

An Algorithmic Exploration Of Dominating Set And Independent Dominating Set

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ABSTRACT

Objectives: This paper presents an algorithmic investigation into the identification and analysis of dominating sets and independent dominating sets in simple graphs. **Methods:** A dominating set and an independent dominating set are found in the following definitions (i) A set $S \subseteq V$ is a dominating set of a graph G if every vertex $v \in V$ is either an element of S or is adjacent to an element of S . (ii) An independent dominating set of G is a set that satisfies both dominating and independent in G . **Findings:** The possibilities of a dominating set and an independent dominating set are found in Python programming language. **Novelty:** The Python programming language to implement algorithms that precisely identify dominating sets and independent dominating sets in simple graphs.

Keywords: Domination – Independent Domination- python program code.

INTRODUCTION

The development of algorithms provides systematic and efficient solutions to a wide range of problems across different domains. Algorithms are a fundamental concept in computer science and critical for advancing technology and solving complex challenges. Through the paper, the graph $G = (V, E)$ means a simple, finite, undirected, and connected graph. A set $S \subseteq V$ is a dominating set of a graph G if every vertex $v \in S$ either an element of S or is adjacent to an element of S [4]. In 1958[1], Berge defined the concept of the domination number of a graph, calling this as “coefficient of External Stability”. In 1962[3], Ore used the name “dominating set” and “domination number” for the same concept. In 1977[2] Cockayne and Hedetniemi made an interesting and extensive survey of the results known at that time about dominating sets in graphs. They have used the notation $\gamma(G)$ for the domination number of a graph. The domination number of G , denoted by $\gamma(G)$, is the minimum size of a dominating set. A set is independent (or stable) if no two vertices are adjacent. An independent dominating set of G is a set that is both dominating and independent in G . The independent domination number and the notation $i(G)$ were introduced by Cockayne and Hedetniemi in [5,6]

is the minimum size of an independent dominating set. The independence number of G , denoted $\alpha(G)$, is the maximum size of an independent set in G . It follows immediately that $\gamma(G) \leq i(G) \leq \alpha(G)$. A dominating set of G of size $\gamma(G)$ is called a γ -set, while an independent dominating set of G of size $i(G)$ is called an i -set [7].

2. RESULT AND DISCUSSION

2.1. An algorithm on Dominating set of any simple graph using Python language:

```
class MyGraph:

    def __init__(self, num_vertices):

        self.V = num_vertices

        self.adj = []

        for i in range(num_vertices):

            self.adj.append([])

    def add_my_edge(self, v1, v2):

        self.adj[v1].append(v2)

        self.adj[v2].append(v1)
```

Relevant conflicts of interest/financial disclosures: The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

```
def is_my_dominating(self, dom_set):
```

```
    for i in range(self.V):
```

```
        if i not in dom_set:
```

```
            flag = False
```

```
            for j in self.adj[i]:
```

```
                if j in dom_set:
```

```
                    flag = True
```

```
                    break
```

```
            if not flag:
```

```
                return False
```

```
    for i in dom_set:
```

```
        for j in dom_set:
```

```
            if i != j and j in self.adj[i]:
```

```
                return False
```

```
    return True
```

```
def find_my_dominating_sets(self):
```

```
    all_sets = []
```

```
    for i in range(1 << self.V):
```

```
        dom_set = []
```

```
        for j in range(self.V):
```

```
            if i & (1 << j):
```

```
                dom_set.append(j)
```

```
            if self.is_my_dominating(dom_set):
```

```
                all_sets.append(dom_set)
```

```
    return all_sets
```

```
g = MyGraph(8)
```

```
g.add_my_edge(0, 1)
```

```
g.add_my_edge(0, 7)
```

```
g.add_my_edge(1, 2)
```

```
g.add_my_edge(2, 7)
```

```
g.add_my_edge(2, 3)
```

```
g.add_my_edge(3, 4)
```

```
g.add_my_edge(4, 5)
```

```
g.add_my_edge(4, 6)
```

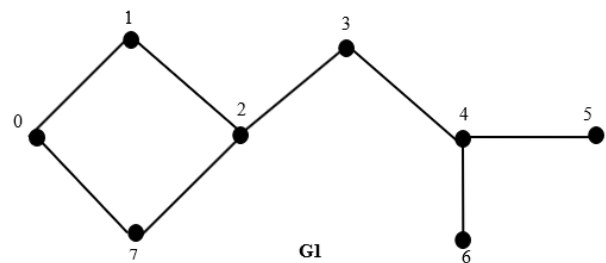
```
dom_sets = g.find_my_dominating_sets()
```

```
print(dom_sets)
```

Output:

```
[[0,2,4], [0,2,5,6], [0,3,5,6], [1,4,7], [1,3,5,6,7]]
```

Manual output:



The possible dominating sets of G1 are: {[0,2,4], [0,2,5,6], [0,3,5,6], [1,4,7], [1,3,5,6,7]}

2.2 An algorithm on Independent Dominating set using Python language

```
class MyGraph:
```

```
    def __init__(self, num_nodes):
```

```
        self.num_nodes = num_nodes
```

```
        self.adj_list = []
```

```
        for node in range(num_nodes):
```

```
            self.adj_list.append([])
```

```
    def add_my_edge(self, node1, node2):
```

```
        self.adj_list[node1].append(node2)
```

```
        self.adj_list[node2].append(node1)
```

```
    def is_my_independent_set(self, my_set):
```

```
        for node1 in my_set:
```

```
            for node2 in my_set:
```

```

        if node1 != node2 and node2 in g.adj_list[node1]:
            return False
        return True

    def is_my_dominating_set(self, my_set):
        covered = my_set.copy()
        for node1 in my_set:
            for node2 in self.adj_list[node1]:
                covered.append(node2)
        covered_set = set(covered)
        return len(covered_set) == self.num_nodes

    def find_my_independent_dominating_sets(self):
        all_sets = []
        for i in range(1 << self.num_nodes):
            my_set = []
            for j in range(self.num_nodes):
                if i & (1 << j):
                    my_set.append(j)
            if self.is_my_independent_set(my_set) and self.is_my_dominating_set(my_set):
                all_sets.append(my_set)
        return all_sets

g = MyGraph(6)
g.add_my_edge(0, 3)
g.add_my_edge(0, 4)
g.add_my_edge(0, 5)
g.add_my_edge(1, 3)
g.add_my_edge(1, 4)
g.add_my_edge(1, 5)
g.add_my_edge(2, 3)

```

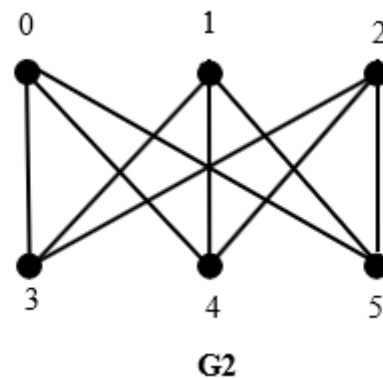
```

g.add_my_edge(2, 4)
g.add_my_edge(2, 5)
ids = g.find_my_independent_dominating_sets()
print(ids)

```

Output:

```
[[0,1,2], [3,4,5]]
```

Manual output:

The possible independent dominating sets of G_2 are:
 $\{[0,1,2], [3,4,5]\}$

CONCLUSION

This paper presents an algorithm developed to determine the possibilities of dominating and independent dominating sets for simple graphs, implemented using Python programming.

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