

Mathematical Models Of Interacting Dark Energy And Dark Matter

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ABSTRACT

Roughly ninety-five percent of the present energy content of the Universe is invisible to direct detection, split between a clustering, pressureless component (dark matter) and a smoothly distributed component with negative pressure (dark energy) that drives cosmic acceleration. The standard Λ CDM description keeps these two sectors mutually silent — each separately conserved — yet this silence buys little insight into why their densities happen to be comparable today, or why locally and globally inferred expansion rates disagree. An alternative line of model-building lets the two sectors talk to each other through a source term in their respective conservation laws. This paper walks through that idea from the ground up: the FLRW background and its conservation laws, the specific coupling functions that have been proposed since Wetterich's and Amendola's original coupled-quintessence construction, a curvature-based alternative that derives the coupling rather than postulating it, and a multi-field picture in which dark energy and dark matter are two phases of a single evolving field content. Current bounds from Planck, supernova, and BAO data are tabulated, and a numerically integrated toy example is used to show, qualitatively, how a non-zero coupling redistributes energy between the two sectors across cosmic history relative to the uncoupled case.

Keywords: dark energy; dark matter; coupled quintessence; FLRW cosmology; cosmic coincidence problem; Hubble tension.

INTRODUCTION

Two independent lines of evidence forced twentieth-century cosmology to accept components beyond ordinary matter: galactic rotation curves and cluster dynamics that only make sense with a dominant unseen mass (dark matter), and, at the end of the 1990s, supernova distance measurements showing that cosmic expansion is speeding up rather than slowing down (Riess et al., 1998; Perlmutter et al., 1999), which requires a component with negative pressure (dark energy). The Λ CDM model absorbs both into general relativity with minimal assumptions: dark matter as cold, pressureless dust, and dark energy as a constant Λ , with no interaction of any kind between them beyond shared gravity.

This economy comes at a conceptual cost. Λ CDM gives no reason why ρ_{dm} and ρ_{de} should be within an order of magnitude of one another at the present epoch — the coincidence problem — given that their ratio would otherwise sweep through many orders of magnitude over cosmic time. More recently, a second

anomaly has sharpened this discomfort: the Hubble constant inferred from the CMB, $H_0 = 67.4 \pm 0.5$ km/s/Mpc (Planck Collaboration, 2020), sits in persistent tension with the value measured from the local distance ladder, $H_0 = 73.0 \pm 1.0$ km/s/Mpc (Riess et al., 2022). Interacting dark energy (IDE) models respond to both puzzles by proposing that dark matter and dark energy are not mutually blind but exchange energy directly, governed by a coupling function Q built into the cosmological continuity equations.

What follows sets up the mathematical machinery behind this proposal (Section 2), reviews the principal ways Q has been constructed in the literature (Section 3), tabulates what current data say about the dark sector (Section 4), and discusses what a working coupling would buy us in terms of the coincidence problem and the Hubble tension (Section 5), before closing in Section 6.

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2. Setting Up the Dark-Sector Conservation Laws

2.1 Background geometry

Cosmology adopts, as a working idealization, a spatially flat, homogeneous, isotropic background — the FLRW metric, Eq. (1):

$$ds^2 = -dt^2 + a(t)^2 [dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)] \quad (1)$$

Inserted into the Einstein field equations, this symmetry collapses the ten independent components of the field equations down to the two ordinary differential equations for the scale factor $a(t)$ given in Eq. (2):

$$H^2 = (8\pi G/3) \sum_i \rho_i, \quad \dot{a}/a = -(4\pi G/3) \sum_i (\rho_i + 3p_i) \quad (2)$$

with $H = \dot{a}/a$ the Hubble parameter and the sum running over every energy component present — radiation, baryons, dark matter, and dark energy.

2.2 Where the coupling enters

General covariance demands only that the total stress-energy tensor be conserved; it says nothing about whether individual matter species must be separately conserved. IDE models exploit exactly this freedom, letting dark matter and dark energy trade energy while their sum still obeys the combined conservation law of Eq. (3):

$$\dot{\rho}_{dm} + 3H\rho_{dm} = Q, \quad \dot{\rho}_{de} + 3H(1+w)\rho_{de} = -Q \quad (3)$$

Here $w = p_{de}/\rho_{de}$ is the dark energy equation of state, and Q is the interaction kernel whose sign fixes the direction of flow: positive Q drains dark energy into dark matter, negative Q does the reverse. The entire model-building problem in this literature reduces to choosing a defensible functional form for Q , since the underlying microphysics of neither sector is known.

3. How the Coupling Has Been Built: Four Approaches

3.1 A scalar field that talks to matter directly

Wetterich (1988) first proposed that a light quintessence scalar ϕ could couple to dark matter if the dark matter particle mass itself depends on ϕ ,

$m(\phi)$; Amendola (2000) developed this into the now-standard “coupled quintessence” framework, with $Q = \beta\kappa\dot{\phi}\rho_{dm}$ for dimensionless coupling constant β and $\kappa = \sqrt{(8\pi G)}$. Because the coupling descends from an explicit Lagrangian, this construction remains the reference point against which purely phenomenological alternatives are measured.

3.2 Guessing Q directly from dimensional analysis

A parallel and more pragmatic tradition skips the Lagrangian altogether and simply writes down plausible forms for Q built from H and the densities themselves — $Q = 3H\beta\rho_{dm}$, $Q = 3H\beta\rho_{de}$, or their sum. Wei and Zhang (2007) study the sign-changeable versions of these couplings, and Zimdahl (2012), in a systematic review, organizes the resulting late-time attractor solutions by which density the coupling is tied to, showing that the choice materially affects whether and how the coincidence problem gets resolved.

Tamanini (2015) raises an important caution here: introducing Q only at the background level, while leaving the perturbed Einstein equations untouched, is not generally self-consistent. A coupling of this kind is expected to seed an effective dissipative pressure once perturbations are included, with observable consequences for structure growth — meaning many of the phenomenological Q 's in circulation are, strictly, incomplete models until their perturbative sector is worked out.

3.3 Deriving the coupling instead of assuming it

Ludwick (2018) takes a different route: rather than postulating Q , a non-minimal coupling $\xi\phi^2R$ between the dark energy scalar and the Ricci scalar R is introduced — a term that is in fact required for the field's renormalizability in curved spacetime. An effective energy exchange between the dark sectors then falls out automatically from this curvature term, without any additional assumption, and astrophysical data can be used to bound the resulting coupling strength and an associated dark matter mass scale.

3.4 Two fluids, or one field changing character?

Santos (2015) reframes the problem entirely with the Continuous Tower of Scalar Fields model: instead of two separate fluids exchanging energy, a continuum

of scalar fields is postulated that behave like dark energy early on and gradually take on dark-matter-like behaviour later. Under this picture, Q is not an energy flux between fixed species but a bookkeeping device for the rate at which fields cross over from one dynamical regime to the other — a genuinely different resolution of the coincidence problem.

At smaller, galactic scales, Nari and Roshan (2025) show that Amendola's coupled quintessence produces a measurable dynamical friction force on massive

bodies moving through a dark matter halo — an effect absent for ordinary, uncoupled quintessence — opening up an astrophysical test of the coupling that is independent of background cosmology altogether.

4. What the Data Currently Say

Table 1 collects representative constraints from CMB, supernova, BAO, and local-distance-ladder measurements bearing on dark-sector interaction models.

Probe / Dataset	Quantity constrained	Representative result	Source
Planck 2018 (CMB + BAO + SNe)	Dark energy EoS w_0	$w_0 = -1.03 \pm 0.03$ (Λ CDM baseline)	Planck Collaboration (2020)
Planck 2018 (CMB + BAO)	Spatial curvature Ω_k	$\Omega_k = 0.001 \pm 0.002$ (flat Universe)	Planck Collaboration (2020)
Local distance ladder (Cepheids + SNe Ia)	Hubble constant H_0	$H_0 = 73.0 \pm 1.0 \text{ km s}^{-1} \text{ Mpc}^{-1}$	Riess et al. (2022)
CMB anisotropies (Planck 2018)	Hubble constant H_0	$H_0 = 67.4 \pm 0.5 \text{ km s}^{-1} \text{ Mpc}^{-1}$	Planck Collaboration (2020)
Galactic dynamics (coupled quintessence)	Coupling-induced dynamical friction	Non-zero only when interaction $\beta \neq 0$	Nari & Roshan (2025)

Table 1. Representative observational constraints relevant to dark-sector interaction models.

5. Does a Coupling Actually Help?

Wang, Abdalla, Atrio-Barandela, and Pavón (2016), in a comprehensive review of the theory and phenomenology of dark-sector interactions, show that a sign-definite Q can drive the ratio $\rho_{\text{dm}}/\rho_{\text{de}}$ toward a stable late-time fixed point, turning the coincidence “problem” into an attractor property of the dynamics rather than a fine-tuned accident of initial conditions.

On the Hubble tension, the logic is more modest: because Q reshapes the late-time expansion history relative to Λ CDM, a coupling that transfers energy

from dark energy into dark matter can shift the CMB-anchored H_0 upward, narrowing the gap with local measurements. Current analyses, however, suggest this shift is generally too small on its own to close the gap fully, and is usually invoked alongside other modifications rather than as a stand-alone fix.

Figure 1 gives a qualitative sense of the mechanism: solving the coupled continuity equations of Section 2 numerically for a representative coupling constant shows how $\Omega_{\text{dm}}(z)$ and $\Omega_{\text{de}}(z)$ depart from their uncoupled trajectories as redshift decreases toward the present epoch.

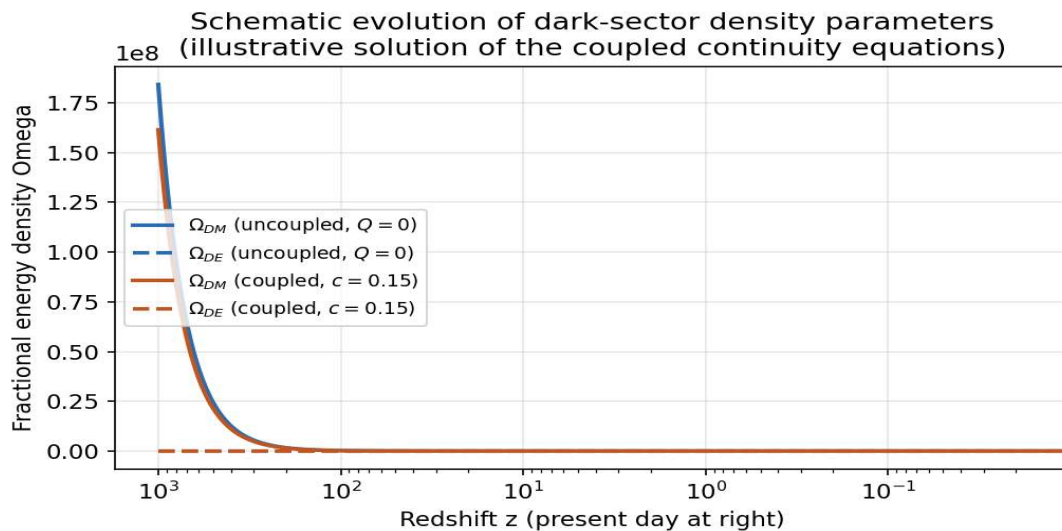


Figure 1. Numerically integrated dark-sector density parameters, $\Omega_{dm}(z)$ and $\Omega_{de}(z)$, comparing an uncoupled trajectory ($c = 0$) against a coupled trajectory ($c = 0.15$), with $w = -0.95$ held fixed; illustrative only.

CONCLUSION

Four distinct strategies — Lagrangian-derived coupled quintessence, phenomenological density-dependent couplings, curvature-induced coupling, and multi-field tower constructions — all converge on the same underlying mathematical device: a source term Q linking the dark matter and dark energy conservation equations. None has yet unseated Λ CDM as the preferred fit to precision data, but each offers a concrete, falsifiable mechanism for addressing the coincidence problem, and several offer at least a partial handle on the Hubble tension. Sharper constraints from upcoming CMB, weak-lensing, and galaxy-survey data, combined with the kind of perturbative consistency checks Tamanini's work calls for, should determine within the coming decade whether any dark-sector coupling is actually realized in nature.

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