Int. J. Sci. R. Tech., 2026 3(8)
A Multidisciplinary peer-reviewed Journal
www.ijsrtjournal.com [ISSN: 2394-7063]

Original Article

Mathematical Techniques To Transform (3, 2)-Fuzzy Bags In Uncertainty Environment

R. Nagarajan[1]*, S. Elango[1], K. Balamurugan[2]

[1]*Department of Mathematics, J.J.College of Engineering and Technology, Tiruchirappalli-620009, Tamilnadu, India.*

[2]*Department of Mathematics, M.A.M School of Engineering, Tiruchirappalli-6221 105, Tamilnadu, India.*

ABSTRACT

In this paper, we introduce the concept of (3,2)-fuzzy multi-groups from (3,2)-fuzzy bags as on extension of intuitionistic fuzzy multi-group, Pythagorean fuzzy multi-group addressing limitation in uncertainty presentation. We develop the theoretical framework for the algebraic structures deriving fundamental properties including closure under group operations and characterizing union and intersection ideas. Key results includes necessary and sufficient conditions for (3,2)-fuzzy uncertainty multi-group properties and comprehensive analysis of their algebraic structures. The theory which helps enhanced tools for decision making under certainty with multiple membership degrees.

Keywords: Multi set, fuzzy bag, (3,2)-fuzzy bag, Multi-group, (3,2)-fuzzy multi-group, beneath, decision making cardinality.

INTRODUCTION

The concept of fuzzy sets was proposed by Zadeh [1]. The theory of fuzzy sets has several applications in real-life situations, and many scholars have researched fuzzy set theory. After the introduction of the concept of fuzzy sets, several research studies were conducted on the generalizations of fuzzy sets. /e integration between fuzzy sets and some uncertainty approaches such as soft sets and rough sets has been discussed in [2–4]. The idea of intuitionistic fuzzy sets suggested by Atanassov [5] is one of the extensions of fuzzy sets with better applicability. Applications of intuitionistic fuzzy sets appear in various fields, including medical diagnosis, optimization problems, and multi criteria decision making [6–8]. Yager [9] offered a new fuzzy set called a Pythagorean fuzzy set, which is the generalization of intuitionistic fuzzy sets. Fermatean fuzzy sets were introduced by Senapati and Yager [10], and they also defined basic operations over the Fermatean fuzzy sets. /e concept of fuzzy topological spaces was introduced by Chang [11]. He studied the topological concepts like continuity and compactness via fuzzy topological spaces. Then, Lowen [12] presented a new type of fuzzy topological spaces. Çoker [13] subsequently initiated a study of intuitionistic fuzzy topological spaces. Recently, Olgun et al. [14] presented the concept of Pythagorean fuzzy topological spaces and Ibrahim [15] defined the concept of Fermatean fuzzy topological spaces. In this paper, we introduce the concept of (3,2)-fuzzy multi-groups from (3,2)-fuzzy bags as on extension of intuitionistic fuzzy multi-group, Pythagorean fuzzy multi-group addressing limitation in uncertainty presentation. We develop the theoretical framework for the algebraic structures deriving fundamental properties including closure under group operations and characterizing union and intersection ideas. Key results includes necessary and sufficient conditions for (3,2)-fuzzy uncertainty multi-group properties and comprehensive analysis of their algebraic structures. The theory which helps enhanced tools for decision making under certainty with multiple membership degrees.

2. Preliminaries:

In this section, we establish the fundamental criteria's and definitions that from theoretical foundation for our investigation. We start with the fundamental concept of multi sets and the multi groups, then

Relevant conflicts of interest/financial disclosures: The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

proceed to uncertainty extension, ultimately leading to the framework necessary for defining (3,2)-uncertainty multi groups.

Definition-2.1: Let 'D' be a set. A bag 'S' drawn from 'D' is represented by a function count

'S' or 'J_S' defined as $J_S: D \to \{0, 1, 2, 3, \dots\}$. For each $d \in D$, $J_S(d)$ is the characteristic value of 'd' in 'S'. Hence $J_S(d)$ denotes the number of possibilities of 'd' in 'S'.

Definition-2.2: Let 'D' be a group. A bag 'S' over 'D' is a multi-group over 'D' if the count of 'S'satisfies the following two axioms,

(i) $J_S(d,f) \geq min\{J_S(d), J_S(f)\}$, for all $d, f \in D$.

(ii) $J_S(d^{-1}) \geq J_S(d)$, for all $d \in D$.

Definition-2.3: If 'D' is a collection of objects, then a fuzzy set 'A' in 'D' is a set of ordered pairs; $D = \{(d, \mu(d))/d \in D, \mu: D \to [0,1]\}$ where μ is called the membership function and is defined from 'D' into $[0,1]$.

Definition-2.4: Assume 'D' is a set of elements, then a fuzzy bag or simply called fuzzy multi set 'S' drawn from 'D' can be characterized by a count membership function JM_S such that $JM_S: D \to R$, where 'R' is the set of all crisp bags from the unit interval [0,1]. A fuzzy bag can also be characterized by high –order function. In particular, a fuzzy bag 'S' can be characterized by a function $JM_S: D \to [0,1] \to$ Natural numbers.

It follows that $JM_S(d)$ for $d \in D$ is given by $JM_S(d) = \{\mu_S^1(d), \mu_S^2(d), \dots, \mu_S^n(d), \dots\}$

where $\mu_S^1(d), \mu_S^2(d), \dots, \mu_S^n(d), \dots \in [0,1]$ such that $\mu_S^1(d) \geq \mu_S^2(d) \geq \cdots \geq \mu_S^n(d) \geq \cdots$.

Where as in a finite case, we write $JM_S(d) = \{\mu_S^1(d), \mu_S^2(d), \dots, \mu_S^n(d)\}$ for $\mu_S^1(d) \geq \mu_S^2(d) \geq \cdots \geq \mu_S^n(d)$.

A fuzzy bag 'S' can be represented by in the form $S = \{((d, JM_S(d))/d \in D)\}$.

Definition-2.5: Let $(D, .)$ be a group. A fuzzy bag 'S' over 'D' is called multi-group over 'D' if the count membership function JM_S satisfies the following conditions

(i) $JM_S(d.f) \geq min\{JM_S(d), JM_S(f)\}$, for all $d, f \in D$.

(ii) $JM_S(d^{-1}) = JM_S(d)$, for all $d \in D$.

3. Basic Concepts of (3, 2)-fuzzy bags

This part establish the fundamental frame work of (3, 2)-fuzzy bags, which helps the theoretical foundation for our subsequent development (3, 2)-fuzzy multi groups.

We begin with basis definition of (3, 2)-uncertainty collections and systematically extend the concept to the bag setting, followed by an analysis of their essential algebraic properties.

Definition-3.1: Let X be a non-empty set. Then a (3, 2)-fuzzy bag 'A' drawn from 'X' is of the form $A = \{(x,\ JM_A(x),\ JN_A(x))/x \in X\}$ where

$JM_A(x) = \alpha_A^1(x), \alpha_A^2(x), \alpha_A^3(x), \dots, \alpha_A^n(x)$ and $JN_A(x) = \beta_A^1(x), \beta_A^2(x), \beta_A^3(x), \dots, \beta_A^n(x)$ are the count membership and count non-membership degrees defined by the function

$JM_A: X \to N^{[0,1]}$ and $JN_A: X \to N^{[0,1]}$ such that $0 \leq (JM_A(x))^3 + (JN_A(x))^2 \leq 1$, where $N = N \cup \{0\}$. For each (3, 2)-fuzzy bag 'A' of 'X'

$$JH_A(x) = \sqrt{1 - \left[(JM_A(x))^3 + (JN_A(x))^2\right]}$$

is the count hesitation margin of x in 'A', where $JH_A(x) = \pi_A^1(x), \pi_A^2(x), \pi_A^3(x), \dots, \pi_A^n(x)$.

The count hesitation margin $JH_A(x)$ is the degree of non-determinacy of $x \in X$ to A and $JH_A(x) \in [0,1]$. The count hesitation margin is the function that expresses lack of knowledge of whether $x \in A$ or $x \notin A$. Thus,

$$(JM_A(x))^3 + (JN_A(x))^2 + (JH_A(x))^5 = 1.$$

We denote the set of all (3, 2)-fuzzy bag over X by (3, 2)-fuzzy bag of X.

Example-3.2: Let 'A' be a (3, 2)-fuzzy bags of $D = \{d_1, d_2, d_3\}$ such that

$$JM_A(d_1) = \{0.23, 0.14\}, \qquad JN_A(d_1) = \{0.78, 0.64\},$$

$$JM_A(d_2) = \{0.62, 0.32, 0.17\}, \qquad JN_A(d_2) = \{0.73, 0.62, 0.56\},$$

$$JM_A(d_3) = \{0.87, 0.67\}, \qquad JN_A(d_3) = \{0.18, 0.58\}.$$

That is,

$$A = \left\{ \begin{matrix} (d_1, (0.23, 0.14), (0.78, 0.64)), (d_2, (0.62, 0.32, 0.17), (0.73, 0.62, 0.56)), \\ (d_3, (0.87, 0.67), (0.18, 0.58)) \end{matrix} \right\}.$$

Then, the Hesitation degree of d_1, d_2, d_3 are

$$JH_A(d_1) = 0.615, 0.767$$

$$JH_A(d_2) = 0.478, 0.763, 0.905$$

$$JH_A(d_3) = 0.556, 0.602$$

Definition-3.3: The cardinality of $JM_A(a)$ or that of $JN_A(a)$ in a (3, 2)-fuzzy bag 'A' is called length of the elements $d \in A$, and is denoted by $L(a\,; A)$.

(i.e) $L(a\,; A) = |JM_A(d)| = |JN_A(d)|$.

When $|JM_A(d)|$ denotes the cardinality of the membership sequence $JM_A(d)$ and $|JN_A(d)|$ that of non-membership sequence $JN_A(d)$.

Definition-3.4: Two (3, 2)-fuzzy bags A_1 and A_2 drawn from a non-empty set 'X' are said to be equivalent, written $A_1 \sim A_2$ if and only if $L(d\,; A_1) = L(d\,; A_2)$.

The following fundamental operations on (3, 2)-fuzzy bags are explored from (Ejegwa) and provide the algebraic frame work for subsequent development.

1. Inclusion : $A \subset G$ if and only if

$$JM_A^3(s) \leq JM_G^3(s) \text{ and } JN_A^2(s) \geq JN_G^2(s), \forall x \in G.$$

2. Equality : $A = G$ if and only if

$$JM_A^3(s) = JM_G^3(s) \text{ and } JN_A^2(s) = JN_G^2(s), \forall x \in G.$$

3. Complement: $A^C = \left\{ \left(s, JN_A^2(s), JM_A^3(s) \right) / s \in G \right\}$.

4. Intersection : $A \cap G = \left\{ \left(s, min\{JM_A^3(s), JM_G^3(s)\}, max\{JN_A^2(s), JN_G^2(s)\} \right) \right\}$.

5. Union : $A \cup G = \left\{ \left(s, max\{JM_A^3(s), JM_G^3(s)\}, min\{JN_A^2(s), JN_G^2(s)\} \right) \right\}$.

Definition-3.5: Let $A \in$(3, 2)-fuzzy bag of S. Then A^{-1} is defined as

$$JM_{A^{-1}}^3(s) = JM_A^3(s^{-1}) \text{ and } JN_{A^{-1}}^2(s) = JN_A^2(s^{-1}).$$

Definition-3.6: Let $A, G \in$(3, 2)-fuzzy bag of S. Then define $A \circ G$ as

$$JM_{A\circ G}(s) = max\left\{ \{ min\{JM_A(u), JM_G(t) / u, t \in S \text{ and } ut = s\} \} \right\},$$

$$JN_{A\circ G}(s) = min\left\{ \{ max\{JN_A(u), JN_G(t) / u, t \in S \text{ and } ut = s\} \} \right\}.$$

Proposition-3.7: Let $A, G, A_i \in$(3, 2)-fuzzy bags. Then the following properties hold;

(i) $(A^{-1})^{-1} = A$

(ii) $A \subseteq G$ implies $A^{-1} \subseteq G^{-1}$

(iii) $(\cup_{i=1}^{n} A_i)^{-1} = \cup_{i=1}^{n} A_i{}^{-1}$

(iv) $(\cap_{i=1}^{n} A_i)^{-1} = \cap_{i=1}^{n} A_i{}^{-1}$

(v) $(A \circ G)^{-1} = G^{-1} \circ A^{-1}$

4. (3, 2)-fuzzy multi groups and its characterizations

In this section, we introduce the central view of (3, 2)-fuzzy multi groups and establish their fundamental algebraic properties. Throughout this section, we assume that $(D, .)$ is a group with identity element 'e' and binary operation'.'.

Definition-4.1: Let $(D, .)$ be a group and $A = \{(d, J_A(d), K_A(d))/d \in D\}$ be a (3, 2)- uncertainty collection over 'D'. Then 'A' is called a (3, 2)-fuzzy subgroup of 'D' if the following conditions are satisfied,

(i) $J_A^3(xy) \geq min\{J_A^3(x), J_A^3(y)\}$ and $K_A^2(xy) \leq max\{K_A^2(x), K_A^2(y)\}$

(ii) $J_A^3(x^{-1}) \geq J_A^3(x)$ and $K_A^2(x^{-1}) \leq K_A^2(x)$, for all $x, y \in D$.

Definition-4.2: Let $(D, .)$ be a group. A (3, 2)-fuzzy bag 'A' over 'D' is called (3, 2)-fuzzy multi group over 'D' if the count membership and count non-membership function satisfies the following axioms;

(i) $JM_A^3(xy) \geq min\{JM_A^3(x), JM_A^3(y)\}$ and $JN_A^2(xy) \leq max\{JN_A^2(x), JN_A^2(y)\}$,

(ii) $JM_A^3(x^{-1}) \geq JM_A^3(x)$ and $JN_A^2(x^{-1}) \leq JN_A^2(x)$, for all $x, y \in D$.

Here $JM_A(x)$ is called count membership function and $JN_A(x)$ is called count non-membership function.

Proposition-4.3: Let $A \in$(3, 2)-fuzzy bag of (D) and $JM_A^3(x^{-1}) \geq JM_A^3(x)$ and

$JN_A^2(x^{-1}) \leq JN_A^2(x)$. Then, $JM_A^3(x^{-1}) = JM_A^3(x)$ and $JN_A^2(x^{-1}) = JN_A^2(x)$.

Proof: From the given condition, we have $JM_A^3(x^{-1}) \geq JM_A^3(x)$.

Additionally, since $JM_A^3(x) = JM_A^3((x^{-1})^{-1}) \geq JM_A^3(x^{-1})$.

By applying the condition to x^{-1}, we get $JM_A^3(x^{-1}) = JM_A^3(x)$.

Similarly, from $JN_A^2(x^{-1}) \leq JN_A^2(x)$ and $JN_A^2(x) = JN_A^2((x^{-1})^{-1}) \leq JN_A^2(x^{-1})$.

We conclude that $JN_A^2(x^{-1}) = JN_A^2(x)$.

Example-4.4: Consider the group $G = Z_3 = \{0, 1, 2\}$ under addition modulo 3. We define a

(3, 2)-fuzzy bag 'A' of G with the following counter membership and count non-membership functions.

$$JM_A^3(0) = \{0.7, 0.6, 0.2\}, \qquad JN_A^2(0) = \{0.1, 0.4, 0.8\}$$

$$JM_A^3(1) = \{0.7, 0.5, 0.3\}, \qquad JN_A^2(1) = \{0.1, 0.5, 0.7\}$$

$$JM_A^3(2) = \{0.7, 0.5, 0.3\}, \qquad JN_A^2(2) = \{0.1, 0.5, 0.7\}$$

We check that 'A' form a (3, 2)-fuzzy multi group by verifying the required conditions;

For all $l, m \in G$,

$$JM_A^3(l+m) \geq min\{JM_A^3(l), JM_A^3(m)\},$$

$$JN_A^2(l+m) \leq max\{JN_A^2(l), JN_A^2(m)\}.$$

Case-1: $0 + 0 = 0$

$$JM_A^3(0) = 0.7, 0.6, 0.2\ , JM_A^3(0) = 0.7, 0.6, 0.2$$

Verified that $JM_A^3(0) \geq 0.7, 0.6, 0.2$.

Similarly,

$$JN_A^2(0) = 0.1, 0.4, 0.8\ , JN_A^2(0) = 0.1, 0.4, 0.8$$

Verified that $JN_A^3(0) \leq 0.1, 0.4, 0.8$.

Case-2: $0 + 1 = 1$

$$JM_A^3(1) = 0.7, 0.5, 0.3\ , min\{JM_A^3(0), JM_A^3(1)\} = 0.7, 0.5, 0.3$$

Verified that $JM_A^3(1) \geq 0.7, 0.5, 0.3$.

Similarly,

$$JN_A^2(1) = 0.1, 0.5, 0.7\ , max\{JN_A^2(0), JN_A^2(1)\} = 0.1, 0.4, 0.7$$

Verified that $JN_A^3(1) \leq 0.1, 0.4, 0.7$.

Case-3: $1 + 2 = 0$

$$JM_A^3(0) = 0.7, 0.6, 0.2\ , min\{JM_A^3(1), JM_A^3(2)\} = 0.7, 0.5, 0.3$$

Verified that $JM_A^3(0) \geq 0.7, 0.5, 0.3$.

Similarly,

$$JN_A^2(0) = 0.1, 0.4, 0.8\ , max\{JN_A^2(1), JN_A^2(2)\} = 0.1, 0.5, 0.7$$

Verified that $JN_A^3(0) \leq 0.1, 0.5, 0.7$.

For all $l \in G$, verify $JM_A^3(-l) \geq JM_A^3(l)$ and $JN_A^2(-l) \leq JN_A^2(l)$.

Case-1: $-0 = 0$

$$JM_A^3(0) = 0.7, 0.6, 0.2, \quad JN_A^2(0) = 0.1, 0.4, 0.8$$

Trivially satisfied.

Case-2: $-1 = 2$

$$JM_A^3(2) = 0.7, 0.5, 0.3, \quad JM_A^3(1) = 0.7, 0.5, 0.3$$

Verified that $JM_A^3(2) \geq JM_A^3(1)$

Similarly, $JN_A^2(2) = 0.1, 0.5, 0.7,\ \ JN_A^2(1) = 0.1, 0.5, 0.7$

Verified that $JN_A^2(2) \leq JN_A^2(1)$.

Case-3: $-2 = 1$

$$JM_A^3(1) = 0.7, 0.5, 0.3, \quad JM_A^3(2) = 0.7, 0.5, 0.3$$

Verified that $JM_A^3(1) \geq JM_A^3(2)$

Similarly, $JN_A^2(1) = 0.1, 0.5, 0.7,\ JN_A^2(2) = 0.1, 0.5, 0.7$

Verified that $JN_A^2(1) \leq JN_A^2(2)$.

Proposition-4.5: Let A ∈(3, 2)-fuzzy multi group of D. Then the following properties hold;

(1) $JM_A^3(e) \geq JM_A^3(d), \forall d \in D$.

(2) $JN_A^2(e) \leq JN_A^2(d), \forall d \in D$.

(3) $JM_A^3(d^n) \geq JM_A^3(d), \forall d \in D$.

(4) $JN_A^2(d^n) \leq JN_A^2(d), \forall d \in D$.

(5) $A = A^{-1}$.

Proof: Let $d \in D$. We establish each property;

1. Since $e = d.d^{-1}$, we have

$$JM_A^3(e) = JM_A^3(d.d^{-1})$$
$$\geq min\{JM_A^3(d), JM_A^3(d^{-1})\}$$
$$\geq min\{JM_A^3(d), JM_A^3(d)\}$$
$$\geq JM_A^3(d).$$

2. Similarly,

$$JN_A^2(e) = JN_A^2(d.d^{-1})$$
$$\leq max\{JN_A^2(d), JN_A^2(d^{-1})\}$$
$$\leq max\{JN_A^2(d), JN_A^2(d)\}$$
$$\leq JN_A^2(d).$$

3. By induction produce on 'n',

$$JM_A^3(d^n) \geq min\{JM_A^3(d^{n-1}), JM_A^3(d)\}$$
$$\geq min\{JM_A^3(d), JM_A^3(d), \dots, JM_A^3(d)\} \text{ (By Recursion)}$$
$$\geq JM_A^3(d).$$

4. The proof for the non-membership function follows similarly.

5. From definition-4.2 and proposition-4.3, we have

$$JM_{A^{-1}}^3(d) = JM_A^3(d^{-1}) = JM_A^3(d)$$
$$JN_{A^{-1}}^2(d) = JN_A^2(d^{-1}) = JN_A^2(d)$$

Therefore, $A = A^{-1}$.

Theorem-4.6: Let A ∈(3, 2)-fuzzy bag of 'D'. Then A ∈(3, 2)-fuzzy multi group of D if and only if, for all $d, u \in D$

$$JM_A^3(d.u^{-1}) \geq min\{JM_A^3(d), JM_A^3(u)\} \text{ and } JN_A^2(d.u^{-1}) \leq max\{JN_A^2(d), JN_A^2(u)\}.$$

Proof: ⇒ The condition is necessary.

Suppose A ∈(3, 2)-fuzzy multi group of D. Then, for any $d, u \in D$.

$$JM_A^3(d.u^{-1}) \geq min\{JM_A^3(d), JM_A^3(u^{-1})\}$$

$$\geq min\{JM_A^3(d), JM_A^3(u)\}$$

where the last inequality follows from the inverse condition.

⇐The condition is sufficient.

Conversely, assume the given condition holds, we need to verify (3, 2)-fuzzy multi group conditions

For the inverse property, setting $u = e$ which gives,

$$JM_A^3(d^{-1}) = JM_A^3(d.e^{-1}) \geq min\{JM_A^3(d), JM_A^3(e)\} = JM_A^3(d).$$

For the closure property, we have,

$$JM_A^3(d.u) = JM_A^3(d.(u^{-1})^{-1}) \geq min\{JM_A^3(d), JM_A^3(u)\}.$$

The non-membership function follow by similarly.

Proposition-4.7: Let A ∈ (3, 2)-fuzzy bag of 'D'. Then A ∈ (3, 2)-fuzzy multi group of D if

$$A \circ A^{-1} \subseteq A.$$

Proof: Assume that $A \circ A^{-1} \subseteq A$.From theorem-4.6 it sufficient to show that

$$JM_A^3(d.u^{-1}) \geq min\{JM_A^3(d), JM_A^3(u)\} \text{ and}$$

$$JN_A^2(d.u^{-1}) \leq max\{JN_A^2(d), JN_A^2(u)\}, \text{ for all } d, u \in D.$$

From the definition of composition and the inclusion assumption

$$JM_A^3(d.u^{-1}) \geq JM_{A\circ A^{-1}}^3(d.u^{-1})$$

$$= \max_{t \in S} min\{JM_A^3(t), JM_{A^{-1}}^3(t^{-1}.d.u^{-1})\}$$

$$\geq min\{JM_A^3(d), JM_A^3(u^{-1})\}; t = d.$$

$$= min\{JM_A^3(d), JM_A^3(u)\} \text{ and}$$

$$JN_A^2(d.u^{-1}) \leq JN_{A\circ A^{-1}}^2(d.u^{-1})$$

$$= \min_{t \in S} max\{JN_A^2(t), JN_{A^{-1}}^2(t^{-1}.d.u^{-1})\}$$

$$\leq max\{JN_A^2(d), JN_A^2(u^{-1})\}; t = d.$$

$$= max\{JN_A^2(d), JN_A^2(u)\}.$$

Theorem-4.8: Let $A, G \in$ (3, 2)-fuzzy multi group of 'D'. Then $A \cap G \in$ (3, 2)-fuzzy multi group of 'D'.

Proof: Let $d, u \in A \cap G \in$ (3, 2)-fuzzy bag of D.

$\Rightarrow d, u \in A$ and $d, u \in G$.

$\Rightarrow JM_A^3(d.u^{-1}) \geq min\{JM_A^3(d), JM_A^3(u^{-1})\}, JM_G^3(d.u^{-1}) \geq min\{JM_G^3(d), JM_G^3(u^{-1})\}$ and

$JN_A^2(d.u^{-1}) \leq max\{JN_A^2(d), JN_A^2(u^{-1})\}, JN_G^2(d.u^{-1}) \leq max\{JN_G^2(d), JN_G^2(u^{-1})\}$.

Now, $JM_{A\cap G}^3(d.u^{-1}) = min\{JM_A^3(d.u^{-1}), JM_G^3(d.u^{-1})\}$

$$\geq min\left\{min\{JM_A^3(d), JM_A^3(u^{-1})\}, \quad min\{JM_G^3(d), JM_G^3(u^{-1})\}\right\}$$

$$\geq min\left\{min\{JM_A^3(d), JM_G^3(d)\}, \quad min\{JM_A^3(u^{-1}), JM_G^3(u^{-1})\}\right\}$$

$$= min\{JM_{A\cap G}^3(d), JM_{A\cap G}^3(u)\}$$

Then, $JM_{A\cap G}^3(d.u^{-1}) \geq min\{JM_{A\cap G}^3(d), JM_{A\cap G}^3(u)\}$.

And, $JN_{A\cap G}^2(d.u^{-1}) = max\{JN_A^2(d.u^{-1}), JN_G^2(d.u^{-1})\}$

$$\leq max\left\{max\{JN_A^2(d), JN_A^2(u^{-1})\}, max\{JN_G^2(d), JN_G^2(u^{-1})\}\right\}$$

$$\leq max\left\{max\{JN_A^2(d), JN_G^2(d)\}, max\{JN_A^2(u^{-1}), JN_G^2(u^{-1})\}\right\}$$

$$\leq max\{JN_{A\cap G}^2(d), JN_{A\cap G}^2(u)\}$$

which implies that, $JN_{A\cap G}^2(d.u^{-1}) \leq max\{JN_{A\cap G}^2(d), JN_{A\cap G}^2(u)\}$.

Then, $A \cap G \in$ (3, 2)-fuzzy multi group of D.

Proposition-4.9: Let $A, G \in$ (3, 2)-fuzzy multi group of 'D'. Then,

(i) $JM_{A\cup G}^3(d^{-1}) \geq JM_{A\cup G}^3(d)$, for all $d \in D$,

(ii) $JN_{A\cup G}^2(d^{-1}) \leq JN_{A\cup G}^2(d)$, for all $d \in D$.

Proof: For any $d \in D$,

$$JM_{A\cup G}^3(d^{-1}) = max\{JM_A^3(d^{-1}), JM_G^3(d^{-1})\}$$

$$\geq max\{JM_A^3(d), JM_G^3(d)\}$$

$$= JM_{A\cup G}^3(d).$$

And,

$$JN_{A\cup G}^2(d^{-1}) = max\{JN_A^2(d^{-1}), JN_G^2(d^{-1})\}$$

$$\leq max\{JN_A^2(d), JN_G^2(d)\}$$

$$= JN_{A\cup G}^2(d).$$

From this, it is clear that if $A, G \in (3, 2)$-fuzzy multi group of 'D', then $A \cup G \in (3, 2)$-fuzzy multi group of 'D' if and only if

$$JM^3_{A\cup G}(d.u^{-1}) \geq min\{JM^3_{A\cup G}(d), JM^3_{A\cup G}(u)\} \text{ and}$$

$$JN^2_{A\cup G}(d.u^{-1}) \leq max\{JN^2_{A\cup G}(d), JN^2_{A\cup G}(u)\}.$$

CONCLUSION

(3, 2)-fuzzy multi groups is introduced from (3, 2)-fuzzy bags and studied some characterizations of this. The length of the (3, 2)-fuzzy bags defined with suitable example. The necessary and sufficient conditions of (3, 2)-fuzzy multi groups is proved with on existence. Many key results and comprehensive analysis is explained.

FUTURE WORK:

One can obtain the similar result in the domain of Hesitant uncertainty collection and Bipolar uncertainty set.

REFERENCES

1. L. A. Zadeh, "Fuzzy sets," Information and Control, vol. 8, no. 3, pp. 338–353, 1965.
2. B. Ahmad and A. Kharal, "On fuzzy soft sets," Advances in Fuzzy Systems, vol. 2009, Article ID 586507, 6 pages, 2009.
3. M. Atef, M. I. Ali, and T. M. Al-shami, "Fuzzy soft coveringbased multi-granulation fuzzy rough sets and their applications," Computational and Applied Mathematics, vol. 40, no. 4, p. 115, 2021.
4. N. Ca˘gman, S. Engino˘glu, and F. Çitak, "Fuzzy soft set theory and its application," Iranian Journal of Fuzzy Systems, vol. 8, no. 3, pp. 137–147, 2011.
5. K. T. Atanassov, "Intuitionistic fuzzy sets," Fuzzy Sets and Systems, vol. 20, no. 1, pp. 87–96, 1986.
6. H. Garg and S. Singh, "A novel triangular interval type-2 intuitionistic fuzzy set and their aggregation operators," Iranian Journal of Fuzzy Systems, vol. 15, pp. 69–93, 2018.
7. H. Garg and K. Kumar, "An advanced study on the similarity measures of intuitionistic fuzzy sets based on the set pair analysis theory and their application in decision making," Soft Computing, vol. 22, no. 15, pp. 4959–4970, 2018.
8. H. Garg and K. Kumar, "Distance measures for connection number sets based on set pair analysis and its applications to decision-making process," Applied Intelligence, vol. 48, no. 10, pp. 3346–3359, 2018.
9. R. R. Yager, "Pythagorean fuzzy subsets," in Proceedings of the 2013 joint IFSA world congress and NAFIPS annual meeting (IFSA/NAFIPS), pp. 57–61, IEEE, Edmonton, Canada, 2013.
10. T. Senapati and R. R. Yager, "Fermatean fuzzy sets," Journal of Ambient Intelligence and Humanized Computing, vol. 11, no. 2, pp. 663–674, 2020.
11. C. L. Chang, "Fuzzy topological spaces," Journal of Mathematical Analysis and Applications, vol. 24, no. 1, pp. 182–190, 1968.
12. R. Lowen, "Fuzzy topological spaces and fuzzy compactness," Journal of Mathematical Analysis and Applications, vol. 56, no. 3, pp. 621–633, 1976.
13. D. Çoker, "An introduction to intuitionistic fuzzy topological spaces," Fuzzy Sets and Systems, vol. 88, no. 1, pp. 81–89, 1997.
14. M. Olgun, M. ¨ Unver, and S¸. Yardımcı, "Pythagorean fuzzy topological spaces," Complex & Intelligent Systems, vol. 5, no. 2, pp. 177–183, 2019.
15. H. Z. Ibrahim, "Fermatean fuzzy topological spaces," Journal of Applied Mathematics and Informatics, 2022, In press.

HOW TO CITE: R. Nagarajan[1]*, S. Elango[1], K. Balamurugan[2], Mathematical Techniques To Transform (3, 2)-Fuzzy Bags In Uncertainty Environment, Int. J. Sci. R. Tech., 2026, 3 (8), 941-949. https://doi.org/10.5281/zenodo.22091782