

# On Generalization Of Generalized Star Generalized Continuous Function In Topological Spaces

X. Josphine Selva Rani\*, T. Shyla Isac Mary

Research Department of Mathematics, Nesamony Memorial Christian College, Marthandam - 629165  
(Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Thirunelveli - 627012), Tamil Nadu, India.

## ABSTRACT

In this paper we defined and study a new class of continuous function namely generalization of generalized star generalized continuous function (briefly  $(gg)^*g$ -continuous function) in topological spaces and further we studied the concept of  $(gg)^*g$  – continuous function and some of its aspects are investigated.

**Keywords:**  $(gg)^*g$  - closed sets,  $(gg)^*g$  - continuous function.

## INTRODUCTION

In this paper our aim is to study a based on  $(gg)^*g$ -continuous function. The family of continuous function plays an important role in topology. Observing these, Csaszar introduced the concept of generalized open sets [10]. In 1970, Levin [29] introduced the concept of generalized closed sets and discussed about the properties of sets, closed and open maps, normal and separation axioms, compactness. In 1982, Malghan [22] introduced and studied the concept of generalized closed maps. In 2017, Basavaraj M. Ittanagi and H.G Govardhana Reddy [3] introduced  $gg$ -closed sets in topological spaces. In 2018, I. Christal Bai and T. Shyla Isac Mary [9] introduced  $(gg)^*$ - closed sets in topological spaces. In 2026, X. Josphine Selva Rani and T. Shyla Isac Mary [20] introduced  $(gg)^*g$  – closed sets in topological spaces. The aim of this paper is to be continue the study of  $(gg)^*g$  – continuous function and analyzed the different aspects.

## PRELIMINARIES

Throughout this paper always means the topological spaces  $(X, \tau)$  and  $(Y, \sigma)$  on which no separation axioms are assumed unless explicitly stated. For a subset  $A$  of a topological space  $(X, \tau)$ , the closure of  $A$  and the interior of  $A$  are denoted by  $cl(A)$  and  $int(A)$

respectively,  $A^c$  denotes the compliment of  $A$  in  $(X, \tau)$ .

### Definition:1

The closure of a subset  $A$  of a topological space  $(X, \tau)$  is the smallest closed set containing  $A$  is denoted by  $cl(A)$ .

The generalized closure (briefly  $g$  - closure) of a subset  $A$  of a topological space  $(X, \tau)$  is the smallest  $g$  - closed set containing  $A$  is denoted by  $gcl(A)$ .

### Definition:2

Generalized - closed set (briefly  $g$  - closed) if  $cl(A) \subseteq U$  [29] whenever  $A \subseteq U$  and  $U$  is open in  $X$ .

Generalization of generalized closed set (briefly  $gg$  - closed) if  $gcl(A) \subseteq U$  [3] whenever  $A \subseteq U$  and  $U$  is regular semi - open in  $X$ .

Generalization of generalized star closed set (briefly  $(gg)^*$  - closed) if  $rcl(A) \subseteq U$  [9] whenever  $A \subseteq U$  and  $U$  is  $gg$  - open in  $X$ .

Generalization of generalized star generalized closed set (briefly  $(gg)^*g$  - closed) if  $gcl(A) \subseteq U$  [20] whenever  $A \subseteq U$  and  $U$  is  $(gg)^*$ - open in  $(X, \tau)$ .

### Definition:3

**Relevant conflicts of interest/financial disclosures:** The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called

- (i) Continuous [30] if the inverse image of every closed set  $V$  in  $(Y, \sigma)$  is closed in  $(X, \tau)$ .
- (ii) regular continuous [31] if the inverse image of every closed set  $V$  in  $(Y, \sigma)$  is regular closed in  $(X, \tau)$ .
- (iii)  $\pi$  - continuous function [14] if the inverse image of every closed set  $V$  in  $(Y, \sigma)$  is  $\pi$  - closed in  $(X, \tau)$ .
- (iv)  $g^*$  - continuous function [32] if the inverse image of every closed set  $V$  in  $(Y, \sigma)$  is  $g^*$  - closed in  $(X, \tau)$ .
- (v)  $(gar)^{**}$  - continuous function [39] if the inverse image of every closed set  $V$  in  $(Y, \sigma)$  is  $(gar)^{**}$  - closed in  $(X, \tau)$ .
- (vi)  $gr^*$  - continuous function [17] if the inverse image of every closed set  $V$  in  $(Y, \sigma)$  is  $gr^*$  - closed in  $(X, \tau)$ .
- (vii)  $(gg)^*$  - continuous function [9] if the inverse image of every closed set  $V$  in  $(Y, \sigma)$  is  $(gg)^*$  - closed in  $(X, \tau)$ .
- (viii)  $r\beta$ - continuous function [23] if the inverse image of every closed set  $V$  in  $(Y, \sigma)$  is  $r\beta$ - closed in  $(X, \tau)$ .
- (ix)  $gp^*$  - continuous function [18] if the inverse image of every closed set  $V$  in  $(Y, \sigma)$  is  $gp^*$  - closed in  $(X, \tau)$ .
- (x)  $(gsp)^*$  - continuous function [33] if the inverse image of every closed set  $V$  in  $(Y, \sigma)$  is  $(gsp)^*$  - closed in  $(X, \tau)$ .
- (xi)  $g^\#$  - continuous function [4] if the inverse image of every closed set  $V$  in  $(Y, \sigma)$  is  $g^\#$  - closed in  $(X, \tau)$ .
- (xii)  $(gs)^*$  - continuous function [1] if the inverse image of every closed set  $V$  in  $(Y, \sigma)$  is  $(gs)^*$  - closed in  $(X, \tau)$ .
- (xiii)  $g^*sr$  - continuous function [43] if the inverse image of every closed set  $V$  in  $(Y, \sigma)$  is  $g^*sr$  - closed in  $(X, \tau)$ .
- (xiv)  $r^*g^*$  - continuous function [27] if the inverse image of every closed set  $V$  in  $(Y, \sigma)$  is  $r^*g^*$  - closed in  $(X, \tau)$ .
- (xv)  $(g^*p)^*$  - continuous function [34] if the inverse image of every closed set  $V$  in  $(Y, \sigma)$  is  $(g^*p)^*$  - closed in  $(X, \tau)$ .
- (xvi)  $r^\wedge g$  - continuous function [37] if the inverse image of every closed set  $V$  in  $(Y, \sigma)$  is  $r^\wedge g$  - closed in  $(X, \tau)$ .
- (xvii)  $rwg$  - continuous function [26] if the inverse image of every closed set  $V$  in  $(Y, \sigma)$  is  $rwg$  - closed in  $(X, \tau)$ .
- (xviii)  $R^*$  - Continuous function [4] if the inverse image of every closed set  $V$  in  $(Y, \sigma)$  is  $R^*$  - closed in  $(X, \tau)$ .
- (xix)  $g^*p$  - Continuous function [38] if the inverse image of every closed set  $V$  in  $(Y, \sigma)$  is  $g^*p$  - closed in  $(X, \tau)$ .
- (xx)  $g^*s^*$  - Continuous function [1] if the inverse image of every closed set  $V$  in  $(Y, \sigma)$  is  $g^*s^*$  - closed in  $(X, \tau)$ .
- (xxi) strongly  $g^*$  - Continuous function [32] if the inverse image of every closed set  $V$  in  $(Y, \sigma)$  is strongly  $g^*$  - closed in  $(X, \tau)$ .
- (xxii)  $g^*s$  - Continuous function [43] if the inverse image of every closed set  $V$  in  $(Y, \sigma)$  is  $g^*s$  - closed in  $(X, \tau)$ .
- (xxiii)  $wg$  - Continuous function [26] if the inverse image of every closed set  $V$  in  $(Y, \sigma)$  is  $wg$  - closed in  $(X, \tau)$ .
- (xxiv)  $pg$  - Continuous function [18] if the inverse image of every closed set  $V$  in  $(Y, \sigma)$  is  $pg$  - closed in  $(X, \tau)$ .
- (xxv)  $ws$  - Continuous function [12] if the inverse image of every closed set  $V$  in  $(Y, \sigma)$  is  $ws$  - closed in  $(X, \tau)$ .
- (xxvi)  $gp$  - Continuous function [13] if the inverse image of every closed set  $V$  in  $(Y, \sigma)$  is  $gp$  - closed in  $(X, \tau)$ .

- (xxvii)  $\alpha g$  - Continuous function [24] if the inverse image of every closed set  $V$  in  $(Y, \sigma)$  is  $\alpha g$  - closed in  $(X, \tau)$ .
- (xxviii)  $gs$  - Continuous function [36] if the inverse image of every closed set  $V$  in  $(Y, \sigma)$  is  $gs$  - closed in  $(X, \tau)$ .
- (xxix)  $gsp$  - Continuous function [33] if the inverse image of every closed set  $V$  in  $(Y, \sigma)$  is  $gsp$  - closed in  $(X, \tau)$ .
- (xxx)  $sg$  - Continuous function [24] if the inverse image of every closed set  $V$  in  $(Y, \sigma)$  is  $sg$  - closed in  $(X, \tau)$ .
- (xxxii)  $g\alpha$  - Continuous function [21] if the inverse image of every closed set  $V$  in  $(Y, \sigma)$  is  $g\alpha$  - closed in  $(X, \tau)$ .
- (xxxiii)  $\alpha$  - Continuous function [42] if the inverse image of every closed set  $V$  in  $(Y, \sigma)$  is  $\alpha$  - closed in  $(X, \tau)$ .
- (xxxiv)  $b$  - Continuous function [11] if the inverse image of every closed set  $V$  in  $(Y, \sigma)$  is  $b$  - closed in  $(X, \tau)$ .
- (xxxv)  $semi$  - Continuous function [8] if the inverse image of every closed set  $V$  in  $(Y, \sigma)$  is  $semi$  - closed in  $(X, \tau)$ .
- (xxxvi)  $gb$  - Continuous function [40] if the inverse image of every closed set  $V$  in  $(Y, \sigma)$  is  $gb$  - closed in  $(X, \tau)$ .
- (xxxvii)  $gb^*$  - Continuous function [45] if the inverse image of every closed set  $V$  in  $(Y, \sigma)$  is  $gb^*$  - closed in  $(X, \tau)$ .
- (xxxviii)  $g^*s$  - Continuous function [1] if the inverse image of every closed set  $V$  in  $(Y, \sigma)$  is  $g^*s$  - closed in  $(X, \tau)$ .

### 1. On Generalization of Generalized star generalized continuous function in Topological Spaces

#### Definition:1.1

A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called  $(gg)^*g$ -continuous if the inverse image of

every closed set  $V$  in  $(Y, \sigma)$  is  $(gg)^*g$  - closed set in  $(X, \tau)$ .

#### Proposition:1.2

Every continuous function is  $(gg)^*g$  - continuous function.

#### Proof:

Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a continuous function and let  $V$  be a closed set in  $(Y, \sigma)$ .

Since  $f$  is continuous, then by Definition 3[i],  $f^{-1}(V)$  is closed in  $(X, \tau)$ . By Proposition 3.3[20], every closed set is  $(gg)^*g$  - closed. So  $f^{-1}(V)$  is  $(gg)^*g$  - closed in  $(X, \tau)$ . By Definition 1.1,  $f$  is  $(gg)^*g$ -continuous function.

**Remark:** The converse part of the above proposition need not be true as shown in the following example.

#### Example:1.3

Let  $X = Y = \{a, b, c, d\}$ ,  $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$ ,  $\sigma = \{\emptyset, \{c, d\}, \{a, c\}, Y\}$

and  $\tau_c = \{\emptyset, \{b, c, d\}, \{a, c, d\}, \{c, d\}, X\}$ . A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is defined by  $f(a) = c, f(b) = a, f(c) = b, f(d) = d$ .  $f$  is  $(gg)^*g$  - continuous function but not continuous function, because for closed set  $V = \{c, d\}$  in  $(Y, \sigma)$ ,  $f^{-1}(V) = \{a, d\}$ ,  $\{a, d\}$  is not closed in  $(X, \tau)$ .

#### Proposition:1.4

Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a function

- (i) Every regular continuous function is  $(gg)^*g$ -continuous function.
- (ii) Every  $\pi$  - continuous function is  $(gg)^*g$ -continuous function.
- (iii) Every  $g^*$  - continuous function is  $(gg)^*g$ -continuous function.
- (iv) Every  $(gar)^{**}$  - continuous function is  $(gg)^*g$ -continuous function.
- (v) Every  $(gr)^*$  - continuous function is  $(gg)^*g$ -continuous function.

- (vi) Every  $(gg)^*$  - continuous function is  $(gg)^*g$  - continuous function.
- (vii) Every  $r\beta$ - continuous function is  $(gg)^*g$ - continuous function.
- (viii) Every  $gp^*$  - continuous function is  $(gg)^*g$ - continuous function.
- (ix) Every  $(gsp)^*$  - continuous function is  $(gg)^*g$ - continuous function.
- (x) Every  $g^\#$  - continuous function is  $(gg)^*g$ - continuous function.
- (xi) Every  $(gs)^*$  - continuous function is  $(gg)^*g$ - continuous function.
- (xii) Every  $g^*sr$  - continuous function is  $(gg)^*g$ - continuous function.
- (xiii) Every  $r^*g^*$  - continuous function is  $(gg)^*g$ - continuous function.
- (xiv) Every  $(g^*p)^*$  - continuous function is  $(gg)^*g$ - continuous function.

**Proof:**

- (i) Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a regular continuous function and let V be a

closed set in  $(X, \tau)$ . Since f is regular continuous function, then by Definition 3[ii],  $f^{-1}(V)$  is regular closed in  $(X, \tau)$ . By Proposition 3.3[20], every regular closed set is  $(gg)^*g$  - closed. Then  $f^{-1}(V)$  is  $(gg)^*g$  - closed in  $(X, \tau)$ . By Definition 1.1, f is  $(gg)^*g$ - continuous function.

- (ii) Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a  $\pi$  - continuous function and let V be a

closed set in  $(X, \tau)$ . Since f is  $\pi$  - continuous function, then by Definition 3[iii],  $f^{-1}(V)$  is a  $\pi$  - closed in  $(X, \tau)$ . By Proposition 3.3[20], every  $\pi$  - closed set is  $(gg)^*g$  - closed. Then  $f^{-1}(V)$  is  $(gg)^*g$  - closed in  $(X, \tau)$ . By Definition 1.1, f is  $(gg)^*g$ - continuous function.

- (iii) Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a  $g^*$ - continuous function and let V be a

closed set in  $(Y, \sigma)$ . Since f is  $g^*$  - continuous, then by Definition 3[iv],  $f^{-1}(V)$  is  $g^*$  - closed in  $(X, \tau)$ . By Proposition 3.3[20], every  $g^*$ - closed set is  $(gg)^*g$  - closed. So  $f^{-1}(V)$  is  $(gg)^*g$  - closed in  $(X, \tau)$ . By Definition 1.1, f is  $(gg)^*g$ - continuous function.

- (iv) Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a  $(gar)^{**}$  - continuous function and let V

be a closed set in  $(X, \sigma)$ . Since f is  $(gar)^{**}$  - continuous, then by Definition 3[v],  $f^{-1}(V)$  is  $(gar)^{**}$  - closed in  $(X, \tau)$ . By Proposition 3.3[20], every  $(gar)^{**}$  - closed set is  $(gg)^*g$  - closed. So  $f^{-1}(V)$  is  $(gg)^*g$  - closed in  $(X, \tau)$ . By Definition 1.1, f is  $(gg)^*g$ - continuous function.

- (v) Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a  $(gr)^*$  - continuous function and let V be a

closed set in  $(Y, \sigma)$ . Since f is  $(gr)^*$  - continuous, then by Definition 3[vi],  $f^{-1}(V)$  is  $(gr)^*$  - closed in  $(X, \tau)$ . By Proposition 3.3[20], every  $(gr)^*$  - closed set is  $(gg)^*g$  - closed. So  $f^{-1}(V)$  is  $(gg)^*g$  - closed in  $(X, \tau)$ . By Definition 1.1, f is  $(gg)^*g$ - continuous function.

- (vi) Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a  $(gg)^*$  - continuous function and let V be a

closed set in  $(Y, \sigma)$ . Since f is  $(gg)^*$  - continuous, then by Definition 3[vii],  $f^{-1}(V)$  is  $(gg)^*$  - closed in  $(X, \tau)$ . By Proposition 3.3[20], every  $(gg)^*$  - closed set is  $(gg)^*g$  - closed. So  $f^{-1}(V)$  is  $(gg)^*g$  - closed in  $(X, \tau)$ . By Definition 1.1, f is  $(gg)^*g$ - continuous function.

- (vii) Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a  $r\beta$  - continuous function and let V be a

closed set in  $(Y, \sigma)$ . Since f is  $r\beta$  - continuous, then by Definition 3[viii],  $f^{-1}(V)$  is  $r\beta$ - closed in  $(X, \tau)$ . By Proposition 3.3[20], every  $r\beta$  - closed set is  $(gg)^*g$  - closed. So  $f^{-1}(V)$  is  $(gg)^*g$  - closed in  $(X, \tau)$ . By Definition 1.1, f is  $(gg)^*g$ - continuous function.

- (viii) Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a  $gp^*$ - continuous function and let V be a

closed set in  $(Y, \sigma)$ . Since f is  $gp^*$ - continuous, then by Definition 3[ix],  $f^{-1}(V)$  is  $gp^*$  - closed in  $(X, \tau)$ .

By Proposition 3.3[20], every  $gp^*$ - closed set is  $(gg)^*g$  - closed. So  $f^{-1}(V)$  is  $(gg)^*g$  - closed in  $(X, \tau)$ . By Definition 1.1,  $f$  is  $(gg)^*g$ - continuous function.

(ix) Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a  $(gsp)^*$ - continuous function and let  $V$  be a

closed set in  $(Y, \sigma)$ . Since  $f$  is  $(gsp)^*$ - continuous, then by Definition 3[x],  $f^{-1}(V)$  is  $(gsp)^*$ - closed in  $(X, \tau)$ . By Proposition 3.3[20], every  $(gsp)^*$ - closed set is  $(gg)^*g$  - closed. So  $f^{-1}(V)$  is  $(gg)^*g$  - closed in  $(X, \tau)$ . By definition 1.1,  $f$  is  $(gg)^*g$ - continuous function.

(x) Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a  $g^\#$ - continuous function and let  $V$  be a

closed set in  $(Y, \sigma)$ . Since  $f$  is  $g^\#$ - continuous, then by Definition 3[xi],  $f^{-1}(V)$  is  $g^\#$ - closed in  $(X, \tau)$ . By Proposition 3.3[20], every  $g^\#$ - closed set is  $(gg)^*g$  - closed. So  $f^{-1}(V)$  is  $(gg)^*g$  - closed in  $(X, \tau)$ . By Definition 1.1,  $f$  is  $(gg)^*g$ - continuous function.

(xi) Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a  $(gs)^*$ - continuous function and let  $V$  be a

closed set in  $(Y, \sigma)$ . Since  $f$  is  $(gs)^*$ - continuous, then by Definition 3[xii],  $f^{-1}(V)$  is  $(gs)^*$ - closed in  $(X, \tau)$ . By Proposition 3.3[20], every  $(gs)^*$ - closed set is  $(gg)^*g$  - closed. So  $f^{-1}(V)$  is  $(gg)^*g$  - closed in  $(X, \tau)$ . By Definition 1.1,  $f$  is  $(gg)^*g$ - continuous function.

(xii) Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a  $g^*sr$ - continuous function and let  $V$  be a

closed set in  $(Y, \sigma)$ . Since  $f$  is  $g^*sr$ - continuous, then by Definition 3[xiii],  $f^{-1}(V)$  is  $g^*sr$ - closed in  $(X, \tau)$ . By Proposition 3.3[20], every  $g^*sr$ - closed set is  $(gg)^*g$  - closed. So  $f^{-1}(V)$  is  $(gg)^*g$  - closed in  $(X, \tau)$ . By Definition 1.1,  $f$  is  $(gg)^*g$ - continuous function.

(xiii) Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a  $r^*g^*$ - continuous function and let  $V$  be a

closed set in  $(Y, \sigma)$ . Since  $f$  is  $r^*g^*$ - continuous, then by Definition 3[xiv],  $f^{-1}(V)$  is  $r^*g^*$ - closed in  $(X, \tau)$ . By proposition 3.3[20], every  $r^*g^*$ - closed set is  $(gg)^*g$  - closed. So  $f^{-1}(V)$  is  $(gg)^*g$  - closed in

$(X, \tau)$ . By Definition 1.1,  $f$  is  $(gg)^*g$ - continuous function.

(xiv) Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a  $(g^*p)^*$ - continuous function and let  $V$  be a

closed set in  $(Y, \sigma)$ . Since  $f$  is  $(g^*p)^*$ - continuous then by Definition 3[xv],  $f^{-1}(V)$  is  $(g^*p)^*$ - closed in  $(X, \tau)$ . By proposition 3.3[20], every  $(g^*p)^*$ - closed set is  $(gg)^*g$  - closed. So  $f^{-1}(V)$  is  $(gg)^*g$  - closed in  $(X, \tau)$ . By Definition 1.1,  $f$  is  $(gg)^*g$ - continuous function.

**Remark:** The converse of the above proposition need not be true as shown in the following examples.

### Example:1.5

Let  $X = Y = \{a, b, c, d\}$ ,  $\tau = \{\emptyset, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, X\}$ ,  $\sigma = \{\emptyset, \{d\}, \{a, c\}, Y\}$

and  $\tau_c = \{\emptyset, \{a, b, d\}, \{a, b, c\}, \{a, b\}, \{b\}, X\}$ . A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is defined by  $f(a) = a, f(b) = d, f(c) = b, f(d) = c$ .  $f$  is  $(gg)^*g$  - continuous function but not regular continuous function, because for closed set  $V = \{a, b, c\}$  in  $(Y, \sigma)$ ,  $f^{-1}(V) = \{a, c, d\}$ ,  $\{a, c, d\}$  is not regular closed in  $(X, \tau)$ .

### Example:1.6

Let  $X = Y = \{a, b, c, d\}$ ,  $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$ ,  $\sigma = \{\emptyset, \{c, d\}, \{a, c\}, Y\}$

and  $\tau_c = \{\emptyset, \{b, c, d\}, \{a, c, d\}, \{c, d\}, X\}$ . An identity function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is defined by  $f(a) = a, f(b) = b, f(c) = c, f(d) = d$ . By  $f$  is  $(gg)^*g$  - continuous function but not  $\pi$  and  $g^*$ - continuous function, because for closed set  $V = \{a, d\}$  in  $(Y, \sigma)$ ,  $f^{-1}(V) = \{a, d\}$ ,  $\{a, d\}$  is not  $\pi$  and  $g^*$ - closed in  $(X, \tau)$ .

### Example:1.7

Let  $X = Y = \{a, b, c, d\}$ ,  $\tau = \{\emptyset, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, X\}$ ,

$\sigma = \{\emptyset, \{a\}, \{a, b\}, \{b, c, d\}, Y\}$  and  $\tau_c = \{\emptyset, \{a, b, d\}, \{a, b, c\}, \{a, b\}, \{b\}, X\}$ .

A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is defined by  $f(a) = b, f(b) = d, f(c) = b, f(d) = c$ .  $f$  is  $(gg)^*g$  - continuous function but not  $(gar)^{**}$ - continuous

function, because for closed set  $V = \{a, b, c\}$  in  $(Y, \sigma)$ ,  $f^{-1}(V) = \{b, c, d\}$ ,  $\{b, c, d\}$  is not  $(gar)^{**}$ -closed in  $(X, \tau)$ .

**Example:1.8**

Let  $X = Y = \{a, b, c\}$ ,  $\tau = \{\varphi, \{b\}, \{a, c\}, X\}$ ,  $\sigma = \{\varphi, \{b\}, Y\}$ ,  $\tau_c = \{\varphi, \{b\}, \{b, c\}, X\}$ .

A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is defined by  $f(a) = b, f(b) = a, f(c) = b, f(c) = c$ . By  $f$  is  $(gg)^*g$ -continuous function but not  $(gr)^*$  and  $r^*g^*$ -continuous function, because for closed set  $V = \{b\}$  in  $(Y, \sigma)$ ,  $f^{-1}(V) = \{a\}$ ,  $\{a\}$  is not  $(gr)^*$  and  $r^*g^*$ -closed in  $(X, \tau)$ .

**Example:1.9**

Let  $X = Y = \{a, b, c, d\}$ ,  $\tau = \{\varphi, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, X\}$ ,  $\sigma = \{\varphi, \{a\}, \{a, b\}, Y\}$

and  $\tau_c = \{\varphi, \{a, b, d\}, \{a, b, c\}, \{a, b\}, \{b\}, X\}$ . A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is defined by  $f(a) = c, f(b) = d, f(c) = a, f(d) = b$ .  $f$  is  $(gg)^*g$ -continuous function but not  $(gg)^*$ -continuous function, because for closed set  $V = \{a, b\}$  in  $(Y, \sigma)$ ,  $f^{-1}(V) = \{c, d\}$ ,  $\{c, d\}$  is not  $(gg)^*$ -closed in  $(X, \tau)$ .

**Example:1.10**

Let  $X = Y = \{a, b, c, d\}$ ,  $\tau = \{\varphi, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{b, c, d\}, X\}$ ,

$\sigma = \{\varphi, \{a\}, \{b\}, Y\}$  and  $\tau_c = \{\varphi, \{a, b, d\}, \{a, b, c\}, \{a, b\}, \{b\}, X\}$ . A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is defined by  $f(a) = b, f(b) = d, f(c) = b, f(d) = c$ .  $f$  is  $(gg)^*g$ -continuous function but not  $r\beta$ -continuous function, because for closed set  $V = \{b\}$  in  $(Y, \sigma)$ ,  $f^{-1}(V) = \{a\}$ ,  $\{a\}$  is not  $r\beta$ -closed in  $(X, \tau)$ .

**Example:1.11**

Let  $X = Y = \{a, b, c, d\}$ ,  $\tau = \{\varphi, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{b, c, d\}, X\}$ ,

$\sigma = \{\varphi, \{a\}, \{d\}, \{c, d\}, Y\}$ ,  $\tau_c = \{\varphi, \{a, b, d\}, \{a, b, c\}, \{a, b\}, \{b\}, X\}$ . A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is defined by  $f(a) = c, f(b) = d, f(c) = b, f(d) = a$ .  $f$  is  $(gg)^*g$ -continuous function but not  $gs^*$ ,  $gp^*$ ,  $g^*sr$  and  $(gsp)^*$ -continuous function, because for closed set  $V = \{c, d\}$

in  $(Y, \sigma)$ ,  $f^{-1}(V) = \{a, b\}$ ,  $\{a, b\}$  is not  $gs^*$ ,  $gp^*$ ,  $g^*sr$  and  $(gsp)^*$ -closed in  $(X, \tau)$ .

**Example:1.12**

Let  $X = Y = \{a, b, c, d\}$ ,  $\tau = \{\varphi, \{b\}, \{c\}, \{b, c\}, \{b, c, d\}, X\}$ ,

$\sigma = \{\varphi, \{a\}, \{c, d\}, \{a, c, d\}, Y\}$ ,  $\tau_c = \{\varphi, \{a, c, d\}, \{a, b, d\}, \{a, d\}, \{a\}, X\}$ . A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is defined by  $f(a) = a, f(b) = d, f(c) = c, f(d) = b$ .  $f$  is  $(gg)^*g$ -continuous function but not  $g^\#$  and  $(g^*p)^*$ -continuous function, because for closed set  $V = \{c, d\}$  in  $(Y, \sigma)$ ,  $f^{-1}(V) = \{a, b\}$ ,  $\{a, b\}$  is not  $g^\#$  and  $(g^*p)^*$ -closed in  $(X, \tau)$ .

**Proposition:1.13**

Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a function

- (i) Every  $(gg)^*g$ -continuous function is  $r^\wedge g$ -continuous.
- (ii) Every  $(gg)^*g$ -continuous function is  $rwg$ -continuous.

**Proof:**

- (i) Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a  $(gg)^*g$ -continuous function and let  $V$  be a

closed set in  $(Y, \sigma)$ . Since  $f$  is  $(gg)^*g$ -continuous, then by Definition 3[xvi],  $f^{-1}(V)$  is  $(gg)^*g$ -closed in  $(X, \tau)$ . By Proposition 3.3[20], every  $(gg)^*g$ -closed set is  $r^\wedge g$ -closed. So  $f^{-1}(V)$  is  $r^\wedge g$ -closed in  $(X, \tau)$ . By Definition 1.1,  $f$  is  $r^\wedge g$ -continuous function.

- (ii) Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a  $(gg)^*g$ -continuous function and let  $V$  be a

closed set in  $(Y, \sigma)$ . Since  $f$  is  $(gg)^*g$ -continuous, then by Definition 3[xvii],  $f^{-1}(V)$  is  $(gg)^*g$ -closed in  $(X, \tau)$ . By Proposition 3.3[20], every  $(gg)^*g$ -closed set is  $rwg$ -closed. So  $f^{-1}(V)$  is  $rwg$ -closed in  $(X, \tau)$ . By Definition 1.1,  $f$  is  $rwg$ -continuous function.

**Remark:** The converse part of the proposition need not be true as shown in the following examples.

**Example:1.14**

Let  $X = Y = \{a, b, c, d\}$ ,  $\tau =$  in  $(Y, \sigma)$ ,  $f^{-1}(V) = \{a, c\}$ ,  $\{a, c\}$  is not  $(gg)^*g$ -closed in  $(X, \tau)$ .  
 $\{\varphi, \{b\}, \{c\}, \{b, c\}, \{b, c, d\}, X\}$ ,

$\sigma = \{\varphi, \{b\}, \{a, b\}, \{a, b, c\}, \{a, c, d\}, Y\}$ ,  $\tau_c =$  A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is defined by  $f(a) = d, f(b) = b, f(c) = a, f(d) = c$ .  
 $\{\varphi, \{a, c, d\}, \{a, b, d\}, \{a, d\}, \{a\}, X\}$ ,

A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is defined by  $f(a) = d, f(b) = b, f(c) = a, f(d) = c$ .  $f$  is  $r^\wedge g$ -continuous function but not  $(gg)^*g$ -continuous function, because for closed set  $V = \{a, b\}$  in  $(Y, \sigma)$ ,  $f^{-1}(V) = \{b, c\}$ ,  $\{b, c\}$  is not  $(gg)^*g$ -closed in  $(X, \tau)$ .  
 $f$  is  $(gg)^*g$ -continuous function but not  $R^*$ -continuous function, because for closed set  $V = \{a, b\}$  in  $(Y, \sigma)$ ,  $f^{-1}(V) = \{b, c\}$ ,  $\{b, c\}$  is not  $R^*$ -closed in  $(X, \tau)$ .

**Example:1.15**

Let  $X = Y = \{a, b, c, d\}$ ,  $\tau =$   
 $\{\varphi, \{b\}, \{c\}, \{b, c\}, \{b, c, d\}, X\}$ ,

$\sigma = \{\varphi, \{a\}, \{a, b\}, \{a, b, c\}, Y\}$ ,  $\tau_c =$   
 $\{\varphi, \{a, c, d\}, \{a, b, d\}, \{a, d\}, \{a\}, X\}$ . A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is defined by  $f(a) = b, f(b) = c, f(c) = d, f(d) = a$ .  $f$  is  $r^\wedge g$ -continuous function but not  $(gg)^*g$ -continuous function, because for closed set  $V = \{a\}$  in  $(Y, \sigma)$ ,  $f^{-1}(V) = \{d\}$ ,  $\{d\}$  is not  $(gg)^*g$ -closed in  $(X, \tau)$ .

**2. Independent set of  $(gg)^*g$  - continuous functions with other continuous function**

The following example shows that the concept of  $(gg)^*g$ -continuous function is independent from  $R^*$ -Continuous,  $g^*p$ -Continuous,  $g^*s^*$ -Continuous, strongly  $g^*$ -Continuous,  $g^*s$ -Continuous,  $wg$ -Continuous,  $pg$ -Continuous,  $ws$ -Continuous,  $gp$ -Continuous,  $ag$ -Continuous,  $gs$ -Continuous,  $gsp$ -Continuous,  $sg$ -Continuous,  $ga$ -Continuous,  $\alpha$ -Continuous,  $b$ -Continuous, *semi*-Continuous,  $gb$ -Continuous,  $gb^*$ -Continuous,  $g^\wedge s$ -Continuous.

**Example:2.1**

Let  $X = Y = \{a, b, c, d\}$ ,  $\tau =$   
 $\{\varphi, \{a\}, \{c\}, \{a, c\}, \{c, d\}, \{a, c, d\}, X\}$ ,

$\sigma = \{\varphi, \{b\}, \{a, b\}, \{a, b, c\}, \{a, c, d\}, Y\}$ ,  $\tau_c =$   
 $\{\varphi, \{b, c, d\}, \{a, b, d\}, \{b, d\}, \{a, b\}, \{b\}, X\}$ .

A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is defined by  $f(a) = a, f(b) = c, f(c) = b, f(d) = d$ .

$f$  is  $R^*$ -continuous function but not  $(gg)^*g$ -continuous function, because for closed set  $V = \{a, b\}$

**Example:2.2**

Let  $X = Y = \{a, b, c, d\}$ ,  $\tau =$   
 $\{\varphi, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, X\}$ ,

$\sigma = \{\varphi, \{d\}, \{a, d\}, \{a, c, d\}, Y\}$ ,  $\tau_c =$   
 $\{\varphi, \{a, b, d\}, \{a, b, c\}, \{a, b\}, \{b\}, X\}$ .

An identity function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is defined by  $f(a) = a, f(b) = b, f(c) =$

$c, f(d) = d$ .  $f$  is  $(gg)^*g$ -continuous function but not  $g^*p$ -continuous

function, because for closed set  $V = \{a, c, d\}$  in  $(Y, \sigma)$ ,  $f^{-1}(V) = \{a, c, d\}$ .  $\{a, c, d\}$  is not  $g^*p$ -closed in  $(X, \tau)$ .

A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is defined by  $f(a) = d, f(b) = b, f(c) = c, f(d) = a$ .

$f$  is  $g^*p$ -continuous function but not  $(gg)^*g$ -continuous function, because for closed set  $V = \{d\}$  in  $(Y, \sigma)$ ,  $f^{-1}(V) = \{a\}$ ,  $\{a\}$  is not  $(gg)^*g$ -closed in  $(X, \tau)$ .

**Example:2.3**

Let  $X = Y = \{a, b, c, d\}$ ,  $\tau =$   
 $\{\varphi, \{b\}, \{c\}, \{b, c\}, \{b, c, d\}, X\}$ ,

$\sigma = \{\varphi, \{a\}, \{d\}, \{a, b, c\}, Y\}$ ,  $\tau_c =$   
 $\{\varphi, \{a, c, d\}, \{a, b, d\}, \{a, d\}, \{a\}, X\}$ .

A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is defined by  $f(a) = d, f(b) = a, f(c) = b, f(d) = c$ .

$f$  is  $(gg)^*g$ -continuous function but not  $g^*s^*$ -continuous function, because for closed set  $V = \{a, b, c\}$  in  $(Y, \sigma)$ ,  $f^{-1}(V) = \{b, c, d\}$ .  $\{b, c, d\}$  is not  $g^*s^*$ -closed in  $(X, \tau)$ .

A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is defined by  $f(a) = b, f(b) = d, f(c) = c, f(d) = a$ .

$f$  is  $g^*s^*$  - continuous function but not  $(gg)^*g$  - continuous function, because for closed set  $V = \{d\}$  in  $(Y, \sigma), f^{-1}(V) = \{b\}, \{b\}$  is not  $(gg)^*g$  - closed in  $(X, \tau)$ .

**Example:2.4**

Let  $X = Y = \{a, b, c, d\}, \tau = \{\varphi, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, X\}$ ,

$\sigma = \{\varphi, \{c\}, \{a, b, c\}, Y\}, \tau_c = \{\varphi, \{a, b, d\}, \{a, b, c\}, \{a, b\}, \{b\}, X\}$ .

A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is defined by  $f(a) = a, f(b) = d, f(c) = c, f(d) = b$ .

$f$  is  $(gg)^*g$  - continuous function but not strongly  $g^*$  - continuous function, because for closed set  $V = \{a, b, c\}$  in  $(Y, \sigma), f^{-1}(V) = \{a, c, d\}. \{a, c, d\}$  is not strongly  $g^*$  - closed in  $(X, \tau)$ .

A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is defined by  $f(a) = c, f(b) = d, f(c) = b, f(d) = a$ .

$f$  is strongly  $g^*$  - continuous function but not  $(gg)^*g$  - continuous function, because for closed set  $V = \{c\}$  in  $(Y, \sigma), f^{-1}(V) = \{a\}, \{a\}$  is not  $(gg)^*g$  - closed in  $(X, \tau)$ .

**Example:2.5**

Let  $X = Y = \{a, b, c, d\}, \tau = \{\varphi, \{b\}, \{c\}, \{b, c\}, \{b, c, d\}, X\}$ ,

$\sigma = \{\varphi, \{b\}, \{a, d\}, \{a, b, d\}, Y\}, \tau_c = \{\varphi, \{a, c, d\}, \{a, b, d\}, \{a, d\}, \{a\}, X\}$ .

A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is defined by  $f(a) = a, f(b) = b, f(c) = d, f(d) = c$ .

$f$  is  $(gg)^*g$  - continuous function but not  $g^*s$  - continuous function, because for closed set  $V = \{a, b, d\}$  in  $(Y, \sigma), f^{-1}(V) = \{a, b, c\}. \{a, b, c\}$  is not  $g^*s$  - closed in  $(X, \tau)$ .

A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is defined by  $f(a) = c, f(b) = d, f(c) = b, f(d) = a$ .

$f$  is  $g^*s$  - continuous function but not  $(gg)^*g$  - continuous function, because for closed set  $V = \{b\}$  in

$(Y, \sigma), f^{-1}(V) = \{c\}, \{c\}$  is not  $(gg)^*g$  - closed in  $(X, \tau)$ .

**Example:2.6**

Let  $X = Y = \{a, b, c, d\}, \tau = \{\varphi, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, X\}$ ,

$\sigma = \{\varphi, \{b\}, \{a, c, d\}, Y\}, \tau_c = \{\varphi, \{a, b, d\}, \{a, b, c\}, \{a, b\}, \{b\}, X\}$ .

A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is defined by  $f(a) = b, f(b) = a, f(c) = c, f(d) = d$ .

$f$  is  $(gg)^*g$  - continuous function but not  $wg$  - continuous function, because for closed set  $V = \{a, c, d\}$  in  $(Y, \sigma), f^{-1}(V) = \{b, c, d\}. \{b, c, d\}$  is not  $wg$  - closed in  $(X, \tau)$ .

A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is defined by  $f(a) = b, f(b) = d, f(c) = c, f(d) = a$ .

$f$  is  $wg$  - continuous function but not  $(gg)^*g$  - continuous function, because for closed set  $V = \{b\}$  in  $(Y, \sigma), f^{-1}(V) = \{a\}, \{a\}$  is not  $(gg)^*g$  - closed in  $(X, \tau)$ .

**Example:2.7**

Let  $X = Y = \{a, b, c, d\}, \tau = \{\varphi, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{b, c, d\}, X\}$ ,

$\sigma = \{\varphi, \{d\}, \{a, d\}, Y\}, \tau_c = \{\varphi, \{a, c, d\}, \{a, b, d\}, \{c, d\}, \{a, d\}, \{a\}, X\}$ .

A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is defined by  $f(a) = a, f(b) = d, f(c) = b, f(d) = c$ .

$f$  is  $(gg)^*g$  - continuous function but not  $pg$  - continuous function, because for closed set  $V = \{a, d\}$  in  $(Y, \sigma), f^{-1}(V) = \{a, b\}. \{a, b\}$  is not  $pg$  - closed in  $(X, \tau)$ .

Let  $X = Y = \{a, b, c, d\}, \tau = \{\varphi, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, X\}$ ,

$\sigma = \{\varphi, \{d\}, \{a, d\}, Y\}, \tau_c = \{\varphi, \{a, c, d\}, \{a, b, c\}, \{a, b\}, \{b\}, X\}$ .

A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is defined by  $f(a) = c, f(b) = b, f(c) = b, f(d) = d$ .

$f$  is  $pg$  - continuous function but not  $(gg)^*g$  - continuous function, because for closed set  $V = \{d\}$  in  $(Y, \sigma)$ ,  $f^{-1}(V) = \{d\}$ ,  $\{d\}$  is not  $(gg)^*g$  - closed in  $(X, \tau)$ .

**Example:2.8**

Let  $X = Y = \{a, b, c, d\}$ ,  $\tau = \{\varphi, \{b\}, \{c\}, \{b, c\}, \{b, c, d\}, X\}$ ,  
 $\sigma = \{\varphi, \{a\}\{b, c, d\}, Y\}$ ,  $\tau_c = \{\varphi, \{a, b, d\}, \{a, b, c\}, \{a, b\}, \{b\}, X\}$ .

An identity function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is defined by  $f(a) = a, f(b) = b, f(c) =$

$c, f(d) = d$ .  $f$  is  $(gg)^*g$  - continuous function but not  $ws$  - continuous function, because for closed set  $V = \{b, c, d\}$  in  $(Y, \sigma)$ ,  $f^{-1}(V) = \{b, c, d\}$ .  $\{b, c, d\}$  is not  $ws$  - closed in  $(X, \tau)$ .

A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is defined by  $f(a) = c, f(b) = b, f(c) = a, f(d) = d$ .

$f$  is  $ws$  - continuous function but not  $(gg)^*g$  - continuous function, because for closed set  $V = \{a\}$  in  $(Y, \sigma)$ ,  $f^{-1}(V) = \{c\}$ ,  $\{c\}$  is not  $(gg)^*g$  - closed in  $(X, \tau)$ .

**Example:2.9**

Let  $X = Y = \{a, b, c, d\}$ ,  $\tau = \{\varphi, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{b, c, d\}, X\}$ ,  
 $\sigma = \{\varphi, \{b\}, Y\}$ ,  $\tau_c = \{\varphi, \{a, c, d\}, \{a, b, d\}, \{c, d\}\{a, d\}, \{a\}, X\}$ .

A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is defined by  $f(a) = d, f(b) = a, f(c) = b, f(d) = c$ .

$f$  is  $(gg)^*g$  - continuous function but not  $gp$  - continuous function, because for closed set  $V = \{b\}$  in  $(Y, \sigma)$ ,  $f^{-1}(V) = \{c\}$ .  $\{c\}$  is not  $gp$  - closed in  $(X, \tau)$ .

Let  $X = Y = \{a, b, c, d\}$ ,  $\tau = \{\varphi, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, X\}$ ,

$\sigma = \{\varphi, \{b\}, \{a, d\}, Y\}$ ,  $\tau_c = \{\varphi, \{a, c, d\}, \{a, b, c\}, \{a, b\}, \{b\}, X\}$ .

A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is defined by  $f(a) = b, f(b) = d, f(c) = a, f(d) = c$ .

$f$  is  $gp$  - continuous function but not  $(gg)^*g$  - continuous function, because for closed set  $V = \{b\}$  in  $(Y, \sigma)$ ,  $f^{-1}(V) = \{a\}$ ,  $\{a\}$  is not  $(gg)^*g$  - closed in  $(X, \tau)$ .

**Example:2.10**

Let  $X = Y = \{a, b, c, d\}$ ,  $\tau = \{\varphi, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{b, c, d\}, X\}$ ,  
 $\sigma = \{\varphi, \{b\}, \{d\}, \{b, d\}, \{b, c, d\}, Y\}$ ,  $\tau_c = \{\varphi, \{a, c, d\}, \{a, b, d\}, \{c, d\}\{a, d\}, \{a\}, X\}$ .

A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is defined by  $f(a) = d, f(b) = b, f(c) = c, f(d) = a$ .

$f$  is  $(gg)^*g$  - continuous function but not  $ag$  - continuous function, because for closed set  $V = \{b, d\}$  in  $(Y, \sigma)$ ,  $f^{-1}(V) = \{a, b\}$ .  $\{a, b\}$  is not  $ag$  - closed in  $(X, \tau)$ .

Let  $X = Y = \{a, b, c, d\}$ ,  $\tau = \{\varphi, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, X\}$ ,

$\sigma = \{\varphi, \{b\}, \{b, d\}, Y\}$ ,  $\tau_c = \{\varphi, \{a, c, d\}, \{a, b, c\}, \{a, b\}, \{b\}, X\}$ .

A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is defined by  $f(a) = b, f(b) = a, f(c) = c, f(d) = d$ .

$f$  is  $ag$  - continuous function but not  $(gg)^*g$  - continuous function, because for closed set  $V = \{b\}$  in  $(Y, \sigma)$ ,  $f^{-1}(V) = \{a\}$ ,  $\{a\}$  is not  $(gg)^*g$  - closed in  $(X, \tau)$ .

**Example:2.11**

Let  $X = Y = \{a, b, c, d\}$ ,  $\tau = \{\varphi, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{b, c, d\}, X\}$ ,

$\sigma = \{\varphi, \{b\}, \{d\}, \{b, d\}, Y\}$ ,  $\tau_c = \{\varphi, \{a, c, d\}, \{a, b, d\}, \{c, d\}\{a, d\}, \{a\}, X\}$ .

A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is defined by  $f(a) = d, f(b) = b, f(c) = a, f(d) = c$ .

$f$  is  $(gg)^*g$  - continuous function but not  $gs$  - continuous function, because for closed set  $V = \{b\}$  in  $(Y, \sigma)$ ,  $f^{-1}(V) = \{b\}$ .  $\{b\}$  is not  $gs$  - closed in  $(X, \tau)$ .

A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is defined by  $f(a) = b, f(b) = d, f(c) = c, f(d) = a$ .

f is  $gs$  - continuous function but not  $(gg)^*g$  - continuous function, because for closed set  $V = \{b, d\}$  in  $(Y, \sigma)$ ,  $f^{-1}(V) = \{a, b\}$ ,  $\{a, b\}$  is not  $(gg)^*g$  - closed in  $(X, \tau)$ .

**Example:2.12**

Let  $X = Y = \{a, b, c, d\}$ ,  $\tau = \{\varphi, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{b, c, d\}, X\}$ ,  
 $\sigma = \{\varphi, \{b\}, \{b, c, d\}, Y\}$ ,  $\tau_c = \{\varphi, \{a, c, d\}, \{a, b, d\}, \{c, d\}, \{a, d\}, \{a\}, X\}$ .

A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is defined by  $f(a) = b, f(b) = a, f(c) = c, f(d) = d$ .

f is  $(gg)^*g$  - continuous function but not  $gsp$  - continuous function, because for closed set  $V = \{b\}$  in  $(Y, \sigma)$ ,  $f^{-1}(V) = \{a\}$ .  $\{a\}$  is not  $gsp$  - closed in  $(X, \tau)$ .

Let  $X = Y = \{a, b, c, d\}$ ,  $\tau = \{\varphi, \{b\}, \{c\}, \{b, c\}, \{b, c, d\}, X\}$ ,  
 $\sigma = \{\varphi, \{b\}, \{b, d\}, Y\}$ ,  $\tau_c = \{\varphi, \{a, c, d\}, \{a, b, d\}, \{a, d\}, \{a\}, X\}$ .

A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is defined by  $f(a) = c, f(b) = b, f(c) = a, f(d) = d$ .

f is  $\alpha g$  - continuous function but not  $(gg)^*g$  - continuous function, because for closed set  $V = \{b\}$  in  $(Y, \sigma)$ ,  $f^{-1}(V) = \{b\}$ ,  $\{b\}$  is not  $(gg)^*g$  - closed in  $(X, \tau)$ .

**Example:2.13**

Let  $X = Y = \{a, b, c, d\}$ ,  $\tau = \{\varphi, \{b\}, \{c\}, \{b, c\}, \{b, c, d\}, X\}$ ,  
 $\sigma = \{\varphi, \{a\}, \{a, b, c\}, Y\}$ ,  $\tau_c = \{\varphi, \{a, c, d\}, \{a, b, d\}, \{a, d\}, \{a\}, X\}$ .

An identity function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is defined by  $f(a) = a, f(b) = b, f(c) =$

$c, f(d) = d$ . f is  $(gg)^*g$  - continuous function but not  $sg$ - continuous function, because for closed set  $V = \{a, b, c\}$  in  $(Y, \sigma)$ ,  $f^{-1}(V) = \{a, b, c\}$ .  $\{a, b, c\}$  is not  $sg$  - closed in  $(X, \tau)$ .

A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is defined by  $f(a) = c, f(b) = d, f(c) = a, f(d) = b$ .

f is  $sg$  - continuous function but not  $(gg)^*g$  - continuous function, because for closed set  $V = \{a\}$  in  $(Y, \sigma)$ ,  $f^{-1}(V) = \{c\}$ ,  $\{c\}$  is not  $(gg)^*g$  - closed in  $(X, \tau)$ .

**Example:2.14**

Let  $X = Y = \{a, b, c, d\}$ ,  $\tau = \{\varphi, \{a\}, \{c\}, \{a, c\}, \{c, d\}, \{a, c, d\}, X\}$ ,  
 $\sigma = \{\varphi, \{b\}, \{b, c\}, \{b, c, d\}, Y\}$ ,  $\tau_c = \{\varphi, \{b, c, d\}, \{a, b, d\}, \{b, d\}, \{a, b\}, \{b\}, X\}$ .

A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is defined by  $f(a) = b, f(b) = c, f(c) = a, f(d) = d$ .

f is  $(gg)^*g$  - continuous function but not  $g\alpha$  - continuous function, because for closed set  $V = \{b, c\}$  in  $(Y, \sigma)$ ,  $f^{-1}(V) = \{a, b\}$ .  $\{a, b\}$  is not  $g\alpha$  - closed in  $(X, \tau)$ .

A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is defined by  $f(a) = c, f(b) = d, f(c) = a, f(d) = b$ .

f is  $g\alpha$  - continuous function but not  $(gg)^*g$  - continuous function, because for closed set  $V = \{b\}$  in  $(Y, \sigma)$ ,  $f^{-1}(V) = \{d\}$ ,  $\{d\}$  is not  $(gg)^*g$  - closed in  $(X, \tau)$ .

**Example:2.15**

Let  $X = Y = \{a, b, c\}$ ,  $\tau = \{\varphi, \{b\}, \{a, c\}, X\}$ ,  
 $\sigma = \{\varphi, \{a\}, \{a, b\}, Y\}$ ,  $\tau_c = \{\varphi, \{a, c\}, \{b\}, X\}$ ,

A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is defined by  $f(a) = c, f(b) = b, f(c) = a$ .

f is  $(gg)^*g$  - continuous function but not  $\alpha$  - continuous function, because for closed set  $V = \{a, b\}$  in  $(Y, \sigma)$ ,  $f^{-1}(V) = \{b, c\}$ .  $\{b, c\}$  is not  $\alpha$  - closed in  $(X, \tau)$ .

Let  $X = Y = \{a, b, c, d\}$ ,  $\tau = \{\varphi, \{b\}, \{c\}, \{b, c\}, \{b, c, d\}, X\}$ ,  
 $\sigma = \{\varphi, \{a\}, \{b, d\}, Y\}$ ,  $\tau_c = \{\varphi, \{a, c, d\}, \{a, b, d\}, \{a, d\}, \{a\}, X\}$ .

A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is defined by  $f(a) = d, f(b) = c, f(c) = b, f(d) = a$ .



$f$  is  $\alpha$  - continuous function but not  $(gg)^*g$  - continuous function, because for closed set  $V = \{a\}$  in  $(Y, \sigma)$ ,  $f^{-1}(V) = \{d\}$ ,  $\{d\}$  is not  $(gg)^*g$  - closed in  $(X, \tau)$ .

**Example:2.16**

Let  $X = Y = \{a, b, c, d\}$ ,  $\tau = \{\varphi, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, X\}$ ,  
 $\sigma = \{\varphi, \{b\}, \{a, b\}, \{b, c, d\}, Y\}$ ,  $\tau_c = \{\varphi, \{a, b, d\}, \{a, b, c\}, \{a, b\}, \{b\}, X\}$ .

A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is defined by  $f(a) = b, f(b) = a, f(c) = c, f(d) = d$ .

$f$  is  $(gg)^*g$  - continuous function but not  $b$  - continuous function, because for closed set  $V = \{b, c, d\}$  in  $(Y, \sigma)$ ,  $f^{-1}(V) = \{a, c, d\}$ .  $\{b, c, d\}$  is not  $b$  - closed in  $(X, \tau)$ .

$f$  is  $b$  - continuous function but not  $(gg)^*g$  - continuous function, because for closed set  $V = \{b\}$  in  $(Y, \sigma)$ ,  $f^{-1}(V) = \{a\}$ ,  $\{a\}$  is not  $(gg)^*g$  - closed in  $(X, \tau)$ .

**Example:2.17**

Let  $X = Y = \{a, b, c, d\}$ ,  $\tau = \{\varphi, \{a\}, \{c\}, \{a, c\}, \{c, d\}, \{a, c, d\}, X\}$ ,  
 $\sigma = \{\varphi, \{a\}, \{b, c\}, Y\}$ ,  $\tau_c = \{\varphi, \{b, c, d\}, \{a, b, d\}, \{b, d\}, \{a, b\}, \{b\}, X\}$ .

A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is defined by  $f(a) = a, f(b) = c, f(c) = b, f(d) = d$ .

$f$  is  $(gg)^*g$  - continuous function but not semi continuous function, because for closed set  $V = \{b, c\}$  in  $(Y, \sigma)$ ,  $f^{-1}(V) = \{b, c\}$ .  $\{b, c\}$  is not semi closed in  $(X, \tau)$ .

$f$  is semi continuous function but not  $(gg)^*g$  - continuous function, because for closed set  $V = \{a\}$  in  $(Y, \sigma)$ ,  $f^{-1}(V) = \{a\}$ ,  $\{a\}$  is not  $(gg)^*g$  - closed in  $(X, \tau)$ .

**Example:2.18**

Let  $X = Y = \{a, b, c, d\}$ ,  $\tau = \{\varphi, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, X\}$ ,

$\sigma = \{\varphi, \{b\}, \{b, c, d\}, Y\}$ ,  $\tau_c = \{\varphi, \{a, b, d\}, \{a, b, c\}, \{a, b\}, \{b\}, X\}$ .

A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is defined by  $f(a) = c, f(b) = a, f(c) = b, f(d) = d$ .

$f$  is  $(gg)^*g$  - continuous function but not  $gb^*$  and  $gb$  continuous function, because for closed set  $V = \{b, c, d\}$  in  $(Y, \sigma)$ ,  $f^{-1}(V) = \{a, c, d\}$ .  $\{a, c, d\}$  is not  $gb^*$  and  $gb$  - closed in  $(X, \tau)$ .

A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is defined by  $f(a) = b, f(b) = d, f(c) = c, f(d) = a$ .

$f$  is  $gb^*$  and  $gb$  - continuous function but not  $(gg)^*g$  - continuous function, because for closed set  $V = \{b\}$  in  $(Y, \sigma)$ ,  $f^{-1}(V) = \{c\}$ ,  $\{c\}$  is not  $(gg)^*g$  - closed in  $(X, \tau)$ .

**Example:2.19**

Let  $X = Y = \{a, b, c, d\}$ ,  $\tau = \{\varphi, \{b\}, \{c\}, \{b, c\}, \{b, c, d\}, X\}$ ,  
 $\sigma = \{\varphi, \{a\}, \{a, b, d\}, Y\}$ ,  $\tau_c = \{\varphi, \{a, c, d\}, \{a, b, d\}, \{a, d\}, \{a\}, X\}$ .

A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is defined by  $f(a) = c, f(b) = a, f(c) = b, f(d) = d$ .

$f$  is  $(gg)^*g$  - continuous function but not  $g^{\wedge*s}$  - continuous function, because for closed set  $V = \{a, b, d\}$  in  $(Y, \sigma)$ ,  $f^{-1}(V) = \{b, c, d\}$ .  $\{b, c, d\}$  is not  $g^{\wedge*s}$  - closed in  $(X, \tau)$ .

A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is defined by  $f(a) = c, f(b) = a, f(c) = d, f(d) = b$ .

$f$  is  $g^{\wedge*s}$  - continuous function but not  $(gg)^*g$  - continuous function, because for closed set  $V = \{a\}$  in  $(Y, \sigma)$ ,  $f^{-1}(V) = \{b\}$ ,  $\{b\}$  is not  $(gg)^*g$  - closed in  $(X, \tau)$ .

**Remark:** From the above discussion and known results for the relationship between  $(gg)^*g$  - continuous function and other existing continuous function are established in figure 1.

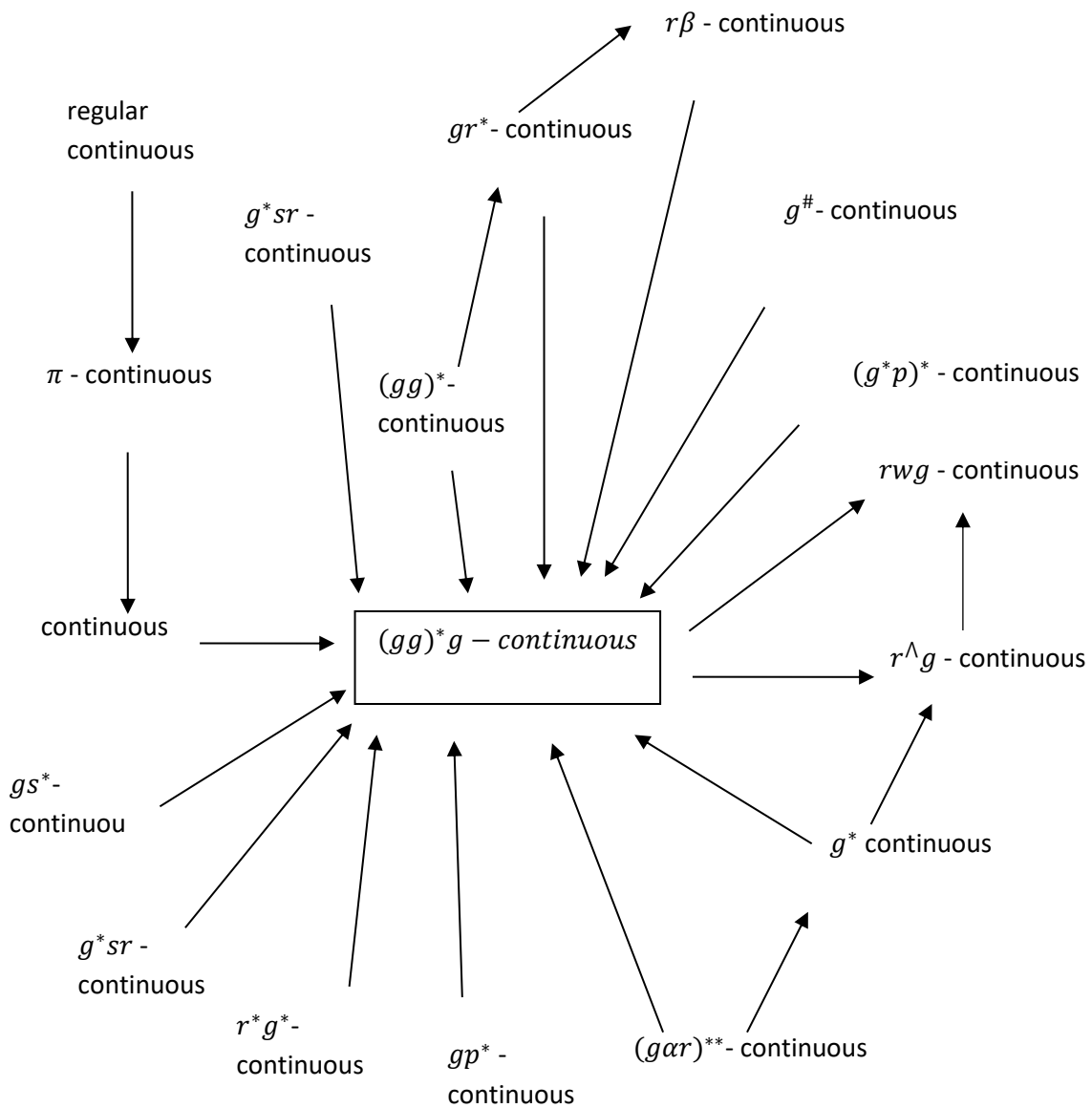


Figure 1

In the Figure 1,  $A \rightarrow B$  means the set A implies B, but not conversely.

In the Figure 2,  $A \leftrightarrow B$  means the set A and B are independent of each other.

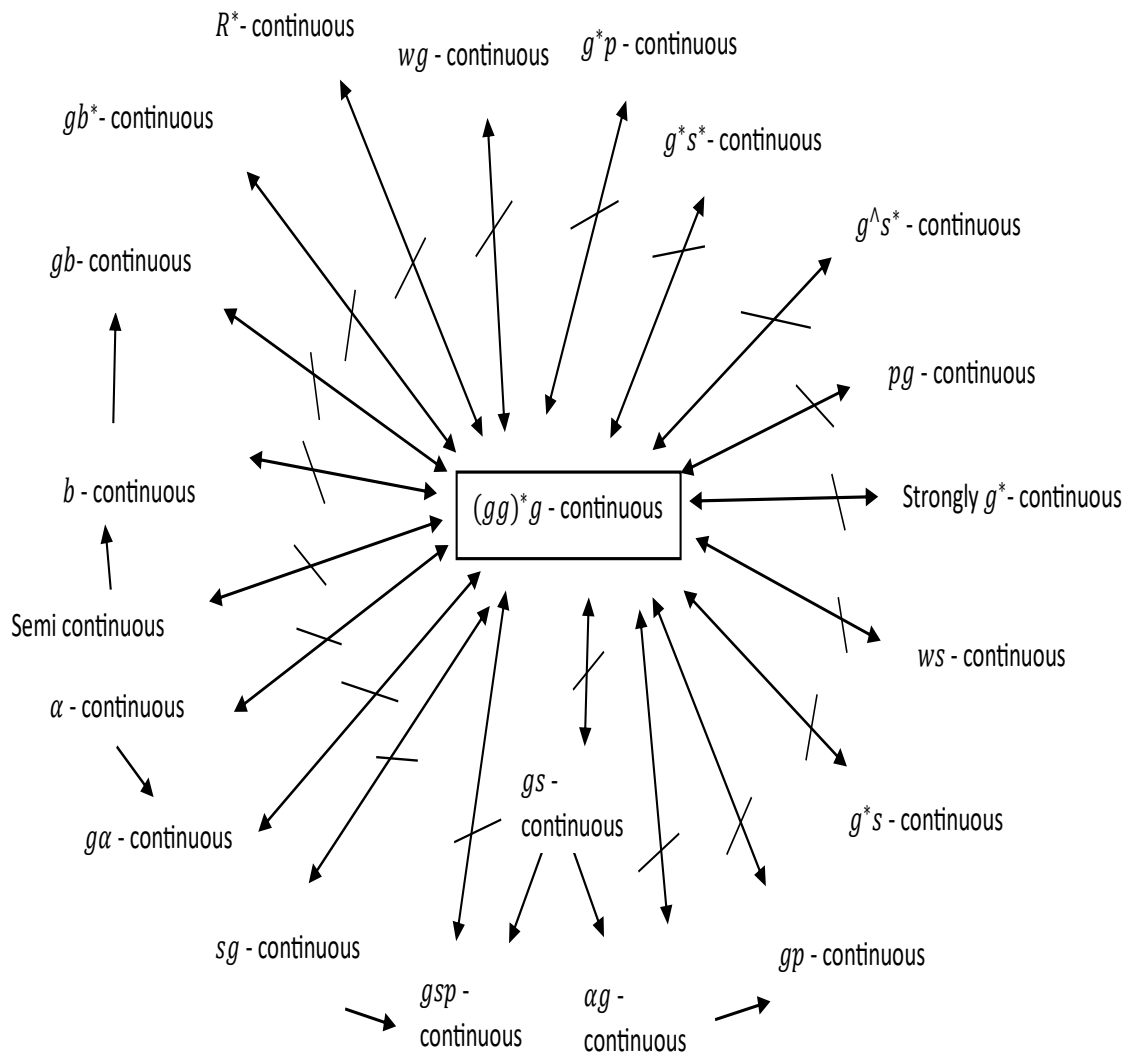


Figure 2

### 3. Aspects of $(gg)^*g$ - continuous function

**Theorem:3.1** If  $f: (X, \tau) \rightarrow (Y, \sigma)$  is  $(gg)^*g$  - continuous function and

$g: (Y, \sigma) \rightarrow (Z, \eta)$  is continuous function then  $(gof): (X, \tau) \rightarrow (Z, \eta)$  is  $(gg)^*g$  - continuous function.

**Proof:**

Let us take U be any closed set in  $(Z, \eta)$ . Since g is continuous function then by

Definition 3[i],  $g^{-1}(U)$  is closed in  $(Y, \sigma)$ . Since f is  $(gg)^*g$  - continuous function then by Definition 1.1,  $f^{-1}(g^{-1}(U))$  is  $(gg)^*g$  - closed set in  $(X, \tau)$ . Therefore  $(gof)^{-1}(U)$  is  $(gg)^*g$  - closed set in  $(X, \tau)$ . Hence  $(gof)$  is  $(gg)^*g$  -continuous function.

**Remark:** Composition of two  $(gg)^*g$  - continuous function need not be  $(gg)^*g$  - continuous function.

**Example:3.2**

Let  $X = Y = Z = \{a, b, c, d\}$ ,  $\tau = \{\emptyset, \{b\}, \{c\}, \{b, c\}, \{b, c, d\}, X\}$

$\sigma = \{\emptyset, \{a\}, \{c\}, \{a, c\}, \{c, d\}, \{a, c, d\}, Y\}$ . A functions  $f: (X, \tau) \rightarrow (Y, \sigma)$  and  $g: (Y, \sigma) \rightarrow (Z, \eta)$  are defined by  $f(a) = a, f(b) = b, f(c) = c, f(d) = d$  and  $g(a) = a, g(b) = b, g(c) = d, g(d) = c$ . f and g are  $(gg)^*g$  - continuous function but not  $(gof)^{-1}$  is  $(gg)^*g$  - continuous function, because for closed set  $V = \{c, d\}$  in  $(Y, \sigma)$ ,  $(gof)^{-1}(V) = \{c, d\}$ .  $\{c, d\}$  is not  $(gg)^*g$  - closed in  $(X, \tau)$ .

**Theorem:3.3**

A function  $f: (X, \tau) \rightarrow (Y, \sigma)$ . Then the following statements are equivalent.

- (i)  $f$  is  $(gg)^*g$  – continuous function.
- (ii)  $f^{-1}(U)$  is  $(gg)^*g$  – open set in  $(X, \tau)$  for every open set  $U$  in  $(Y, \sigma)$ .
- (iii)  $f^{-1}(V)$  is  $(gg)^*g$  – closed set in  $(X, \tau)$  for every closed set  $V$  in  $(Y, \sigma)$ .

**Proof:**

**(i)⇒(ii)** Let  $f$  be a  $(gg)^*g$  – continuous function and  $U$  be an open set in  $(Y, \sigma)$ . Then  $U^c$  is closed set in  $(Y, \sigma)$ . Since  $f$  is  $(gg)^*g$  – continuous function, by Definition 1.1,  $f^{-1}(U^c)$  is  $(gg)^*g$  – closed set in  $(X, \tau)$ . Therefore  $f^{-1}(U^c) = X - f^{-1}(U)$  is  $(gg)^*g$  – closed set in  $(X, \tau)$  and hence  $f^{-1}(U)$  is  $(gg)^*g$  – open set in  $(X, \tau)$ .

**(ii)⇒(iii)** Let us assume that  $f^{-1}(V)$  is  $(gg)^*g$  – open set in  $(X, \tau)$  for every open set  $U$  in  $(Y, \sigma)$ . Let  $V$  be a closed set in  $(Y, \sigma)$ . Then  $V^c$  is open set in  $(Y, \sigma)$ .

By assumption  $f^{-1}(V^c)$  is  $(gg)^*g$  – open set in  $(X, \tau)$ . Therefore  $f^{-1}(V^c) = X - f^{-1}(V)$  is  $(gg)^*g$  – open set in  $(X, \tau)$  and hence  $f^{-1}(V)$  is  $(gg)^*g$  – closed set in  $(X, \tau)$ .

**(ii)⇒(iii)** Let  $f$  be a  $(gg)^*g$  – continuous function and let  $V$  be a closed set in  $(Y, \sigma)$ . Then  $V^c$  is open set in  $(Y, \sigma)$ . Since  $f$  is  $(gg)^*g$  – continuous function, by Definition 1.1,  $f^{-1}(V^c) = X - f^{-1}(V)$  is  $(gg)^*g$  – open set in  $(X, \tau)$ . Hence  $f^{-1}(V)$  is  $(gg)^*g$  – closed set in  $(X, \tau)$ .

**(iii)⇒(i)** Let us assume that  $f^{-1}(V)$  is  $(gg)^*g$  – closed set in  $(X, \tau)$  for every closed set  $V$  in  $(Y, \sigma)$ . Let  $W$  be an open set in  $(Y, \sigma)$ , then  $W^c$  is closed set in  $(Y, \sigma)$ . By assumption  $f^{-1}(W^c)$  is  $(gg)^*g$  – closed set in  $(X, \tau)$ . Therefore  $f^{-1}(W^c) = X - f^{-1}(W)$  is  $(gg)^*g$  – closed set in  $(X, \tau)$  and hence  $f$  is  $(gg)^*g$  – continuous function.

**Theorem:3.4**

Let  $(X, \tau)$  be a topological space in which every singleton set is  $(gg)^*$  – closed.

Then the function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is  $(gg)^*g$  – continuous if  $x \in gint(f^{-1}(V))$  for every open subset  $V$  of  $(Y, \sigma)$  containing  $f(x)$ .

**Proof:**

Assume that a function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is  $(gg)^*g$  – continuous function.

To prove  $x \in gint(f^{-1}(V))$ . Let  $x \in X$  and  $V$  be an open set in  $(Y, \sigma)$  containing  $f(x)$ . That is  $f(x) \in V$ . Since  $f$  is  $(gg)^*g$  – continuous function, by Theorem 3.3  $f^{-1}(V)$  is  $(gg)^*g$  – open set in  $(X, \tau)$  and since  $\{x\}$  is  $(gg)^*$  – closed, then  $x \in gint(f^{-1}(V))$ .

Conversely assume that  $x \in gint(f^{-1}(V))$  for every open subset  $V$  of  $(Y, \sigma)$  containing  $f(x)$ . To prove  $f$  is  $(gg)^*g$  – continuous function.

Let  $V$  be an open set in  $(Y, \sigma)$ . Suppose that  $G \subseteq f^{-1}(V)$  and  $G$  is  $(gg)^*g$  – closed. Let  $x \in G \subseteq f^{-1}(V)$

$$\Rightarrow x \in f^{-1}(V)$$

$$\Rightarrow f(x) \in V$$

By hypothesis,  $x \in gint(f^{-1}(V))$  and from the assumption  $f^{-1}(V)$  is an  $(gg)^*g$  – open set in  $(X, \tau)$ . By Theorem 3.3,  $f$  is  $(gg)^*g$  – continuous function.

**Theorem:3.5**

If  $f$  is  $(gg)^*g$  – continuous function then  $f((gg)^*g cl(A)) \subseteq cl(f(A))$  for every subset  $A$  of  $(X, \tau)$ .

**Proof:**

Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  is  $(gg)^*g$  – continuous function and let  $A \subseteq X$  then  $cl(f(A))$

is closed in  $(Y, \sigma)$ .

Since  $f(A) \subseteq cl(f(A))$ ,  $A \subseteq f^{-1}(cl(f(A)))$

We know that  $cl(f(A))$  is closed in  $(Y, \sigma)$  and also  $f$  is  $(gg)^*g$  – continuous function.

By Definition 1.1,  $f^{-1}(cl(f(A)))$  is  $(gg)^*g$  – closed in  $(X, \tau)$ .

Let us assume that  $y \in f^{-1}((gg)^*g - cl(A))$ . Then  $y = f(x)$ , where  $x \in (gg)^*g - cl(A)$ . Let  $G$  be an

open set containing  $y = f(x)$ . Since  $f$  is  $(gg)^*g -$  continuous function, by Theorem 3.3,  $f^{-1}(G)$  is  $(gg)^*g -$  open set in  $(X, \tau)$ . By Theorem 5.8[20],  $f^{-1}(G) \cap A \neq \varnothing$ .

$$\begin{aligned} &\Rightarrow f(f^{-1}(G) \cap A) \neq \varnothing \\ &\Rightarrow f(f^{-1}(G)) \cap f(A) \neq \varnothing. \\ &f(f^{-1}(G)) \in G \\ &\Rightarrow G \cap f(A) \neq \varnothing. \end{aligned}$$

Therefore  $y \in cl(f(A))$  because for any open set  $G$  containing  $y$ ,  $G \cap f(A) \neq \varnothing$  is a characterization of  $y$  being in the closure of  $f(A)$ . Hence  $f((gg)^*g - cl(A)) \subseteq cl(f(A))$ .

**Theorem:3.6**

Let a function  $f: (X, \tau) \rightarrow (Y, \sigma)$  then the following statements are equivalent.

- (i) For every point  $x \in (X, \tau)$  and each open set  $V$  containing  $f(x)$  in  $(Y, \sigma)$ , there is an

$(gg)^*g -$  open set  $U$  in  $(X, \tau)$  such that  $x \in U$  and  $f(U) \subseteq V$ .

- (ii) For each  $A \subseteq (X, \tau)$ ,  $f((gg)^*g - cl(A)) \subseteq cl f(A)$ .

- (iii) For each  $B \subseteq (X, \tau)$ ,  $(gg)^*gcl(f^{-1}(B)) \subseteq f^{-1}(cl(B))$ .

**Proof:**

**(i)⇒(ii)** Suppose (i) is hold. Let us assume that  $y \in f((gg)^*g - cl(A))$ . Then there exists an element  $x \in (gg)^*g - cl(A)$  such that  $y = f(x)$ . Let  $V$  be an open set containing  $y$  that is  $y \in V, f(x) \in V$ . Since  $x \in (gg)^*gcl(A)$ , by Theorem 5.8[20], there exists an  $(gg)^*g -$  open set  $U$  containing a point  $x$  such that  $U \cap A \neq \varnothing$ .

$$\begin{aligned} &\Rightarrow f(U \cap A) \neq \varnothing \\ &\Rightarrow f(U) \cap f(A) \neq \varnothing \end{aligned}$$

By hypothesis  $f(U) \subseteq V$

$$\Rightarrow f(U) \cap f(A) \subseteq V \cap f(A) \neq \varnothing$$

$$\Rightarrow V \cap f(A) \neq \varnothing$$

Thus  $y \in cl(f(A))$ , because for any open set  $V$  containing  $y$ ,  $V \cap f(A) \neq \varnothing$  is a characterization of  $y$  being in the closure of  $f(A)$ .

$$\text{Therefore } f((gg)^*g - cl(A)) \subseteq cl(f(A)).$$

**(ii)⇒(i)** Suppose (ii) is hold. Let  $V$  be an open set containing  $f(x)$  and let  $x \in (X, \tau)$ . And let  $A = f^{-1}(V^c)$ . Since  $f((gg)^*g - cl(A)) \subseteq cl(f(A))$

$$\begin{aligned} &= cl((V^c)) \\ &= V^c \end{aligned}$$

$$\text{Therefore } f((gg)^*gcl(A)) \subseteq V^c$$

$$\begin{aligned} &\Rightarrow (gg)^*gcl(A) \subseteq f^{-1}(V^c) = A \\ &\Rightarrow (gg)^*gcl(A) \subseteq A \end{aligned}$$

Also we know that  $A \subseteq (gg)^*gcl(A)$  that implies  $(gg)^*gcl(A) = A$ .

$$\text{Since } f(x) \in V, x \in f^{-1}(V)$$

This  $x \notin A$  then  $x \notin (gg)^*gcl(A)$

By Theorem 5.8[20], there exists  $(gg)^*g$  open set  $U$  containing  $x$  such that  $U \cap A \neq \varnothing$

$$\text{That implies } V \subseteq A^c \Rightarrow f(U) \subseteq f(A^c) \subseteq V.$$

Hence  $f(U) \subseteq V$ .

**(ii)⇒(iii)** Suppose (ii) is hold. Let  $B \subseteq (Y, \sigma)$  and replacing  $A$  by  $f^{-1}(B)$  in (ii).

$$\text{We get } f((gg)^*gcl(f^{-1}(B))) \subseteq cl(f(f^{-1}(B))) = cl(B)$$

$$\Rightarrow f((gg)^*gcl(f^{-1}(B))) \subseteq cl(B)$$

$$\Rightarrow (gg)^*gcl(f^{-1}(B)) \subseteq f^{-1}(cl(B))$$

**(iii)⇒(ii)** Suppose (iii) is hold. Let  $A \subseteq (X, \sigma)$  and take  $f(A) = B$  in (iii)

$$\text{We get } (gg)^*gcl(f^{-1}(f(A))) \subseteq f^{-1}(cl(f(A)))$$

$$\Rightarrow (gg)^*gcl(A) \subseteq f^{-1}(cl(f(A)))$$

$$\Rightarrow f((gg)^*gcl(A)) \subseteq cl(f(A)).$$



**Theorem:3.7**

If  $f: (X, \tau) \rightarrow (Y, \sigma)$  is closed and  $(gg)^*g$  - continuous function and B is  $(gg)^*g$  -

closed set of  $(y, \sigma)$  then  $f^{-1}(B)$  is  $(gg)^*g$  - closed set in  $(x, \tau)$ .

**Proof:**

Let B be  $(gg)^*g$  - closed set of  $(Y, \sigma)$  and let  $f^{-1}(B) \subseteq U$ , where U is an open set of

$(X, \tau)$ . Since f is closed, there is an open set V such that  $B \subseteq V$  and  $f^{-1}(B) \subseteq U$ . Since B is  $(gg)^*g$  - closed set  $gcl(B) \subseteq U$ , that implies  $f^{-1}(gcl(B)) \subseteq U$ . Since  $gcl(B) \subseteq cl(B)$ ,

by assumption f is closed and  $(gg)^*g$  - continuous,  $f^{-1}(cl(B))$  is  $(gg)^*g$  - closed set in  $(X, \tau)$ .

$$\Rightarrow gcl(f^{-1}(cl(B))) \subseteq U$$

$$\Rightarrow gcl(f^{-1}(B)) \subseteq U.$$

Therefore  $f^{-1}(B)$  is  $(gg)^*g$  - closed set in  $(X, \tau)$ .

**Theorem:3.8**

If  $f: (X, \tau) \rightarrow (Y, \sigma)$  is  $(gg)^*g$  - continuous function, closed and

$g: (Y, \sigma) \rightarrow (Z, \eta)$  is  $(gg)^*g$  - continuous function then  $gof: (X, \tau) \rightarrow (Z, \eta)$  is  $(gg)^*g$  - continuous function.

**Proof:**

Let us take U be any closed set in  $(Z, \eta)$ . Then  $f^{-1}(U)$  is closed set in  $(Y, \sigma)$ .

Since g is  $(gg)^*g$  - continuous function and closed then  $g^{-1}(U)$  is closed in  $(Y, \sigma)$  and also f is  $(gg)^*g$  - continuous function  $f^{-1}(g^{-1}(U))$  is  $(gg)^*g$  - closed set in  $(X, \tau)$ . Hence  $gof$  is  $(gg)^*g$  - continuous function.

**Theorem:3.9**

In extremely disconnected space  $(X, \tau)$ , If  $f: (X, \tau) \rightarrow (Y, \sigma)$  is g - continuous function and open then  $(gg)^*g$  - continuous function.

**Proof:**

Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a g-continuous function and let V be a

closed set in  $(X, \tau)$ . Since f is g-continuous function and open, then  $f^{-1}(V)$  is g-closed in  $(X, \tau)$ . By Theorem 5.4[20],  $f^{-1}(V)$  is  $(gg)^*g$  - closed, then  $f^{-1}(V)$  is  $(gg)^*g$  - closed in  $(X, \tau)$ . By Definition 1.1, f is  $(gg)^*g$ - continuous function.

**CONCLUSION**

The study of  $(gg)^*g$  - continuous function in topological spaces literary investigated. The concept of  $(gg)^*g$  - continuous function will be extended to strongly  $(gg)^*g$  - continuous function, perfectly  $(gg)^*g$  - continuous function, contra  $(gg)^*g$  - continuous function. Also this study can be elaborated to bitopological spaces and fuzzy topological spaces.

**REFERENCES**

1. Anto M & Andrin Shahila S,  $g^{**}$  s – Closed Set in Topological Spaces, IJMTT-ICIMCEH 2020.
2. Ahmad Al-Omari and Mohd Salmi Md Noorani, Regular Generalized w-Closed Sets, International Journal of Mathematics and Mathematical Sciences, Volume 2007, Article ID 16292.
3. Basavaraj M. Ittanagi and Govardhana Reddy H.G / On gg-Closed Sets in Topological Spaces / IJMA-8(8), August 2017.
4. Basavaraj M. Ittanagi and Raghavendra K / On  $R^\#$ -Closed Sets in Topological Spaces/ IJMA-8(8), August 2017.
5. Benchalli S.S & R.S Wali, On RW - Closed Sets in Topological Spaces, Bull. Malay. Math. Sci. Soc. (2) 30(2) (2007), 99-110.
6. Benchalli S.S and J.B Toranagatti, Delta Generalized Pre-Closed Sets in Topological Spaces; IJCMS: Vol. 11, 2016, no. 6, 281-292.
7. Bhattacharya S(Halder), On Generalized Regular Closed Sets, Int.J. Contemp. Math, Sciences, Vol.6, 2011, no.3, 145 – 152.



8. Cameron D.E, Properties of semi closed Spaces Proc, Amer – Math – Soc, 72, 581-586, 1978.
9. Christal Bai I, T.Shyla Isac Mary/On  $(gg)^*$  - Closed Sets in Topological Spaces / IJSR in MSS, S(4)(2018), 395-403.
10. Csaszar A (2005), Generalized open sets in generalized topologies, Act a Math. Hungar., 106, 53-66.
11. Dhanya R, Parvathi A / on  $\pi gb^*$ -closed sets in topological spaces / IJIRSET / www.ijirset.com,ISSN:2319-8753.(An ISO 3297: 2007 certified organization) vol. 3, Issue 5, May 2014.
12. Dhana Lekshmi D, T. Shyla Isac Mary,  $w^*s$  - Closed Sets in Topological Spaces, Journal of Harbin Engineering University, ISSN:1006-7043, Vol 44, No. 8 August 2023.
13. DillyRani C, Generalized Pre-Closed Set in Topological Spaces, IJRES, www.ijres.org Volume 09 Issue 12\2021\PP. 19-23.
14. Dontchev J and T.Noiri, Quasi – Normal Spaces and  $\pi g$  - Closed Sets, Acta Math. Hungar 89(3) (2000), 211-219
15. Govindappa Navalagi, Chandrashekarappa A.S and S.V. Gurushantanavar, On GSPR – Closed Sets in Topological Spaces, IJMCA, Vol. 2, No. 1-2, January – December 2010, PP.51-58.
16. Govindappa Navalagi and Kantappa. M. Bhavikatti, Beta Weakly Generalized Closed Sets in Topology, JCMS, Vol. 9(5), May 2018, www.compmath-journal.org, 435-446.
17. Indirani K, P. Satishmohan and V. Rajendran, On  $gr^*$  - Closed Sets in Topological Spaces, International Journal of Mathematics Trends and Technology – volume 6 - February 2014.
18. Jayakumar et al P, On Generalized  $gp^*$  - Closed Set in Topological Spaces / Int.Journal of Math. Analysis, Vol. 7, 2013, no. 33, 1635-1645.
19. Jilling Cao, Maximilian Ganster, Ivan Reilly, On Generalized Closed Sets, J.Cao et al./Topology and its applications 123 (2002) 37 – 46.
20. Josphine Selva Rani X., Shyla Isac Mary T., On Generalization of Generalized Star Generalized Closed Sets in Topological Spaces, Bol. Soc. Paran. Mat.(3s) v. 2026 (44):1-16, SPM-ISSN-0037-8712, SPM: www.spm.uem.br/bspm, ©-E-ISSN-2175-1188.
21. Maki H, R. Devi and K. Balachandran, Generalized Alpha Closed Sets in Topology, Bull. Fukuoka Univ. Ed. Part (2),42, (1993) 13-21.
22. Malghan S.R, Generalized closed maps, J. Karnataka Univ. Sci., 27(1982), 82-88.
23. Manonmani A, S. Jayalakshmi, On Regular Beta ( $r\beta$ ) - Closed Sets in Topological Spaces, Research Gate Impact factor (2018): 0.28/ SJIF (2018): 7.426.
24. Meena K & V. P Anuja On Alpha  $\wedge$  Generalized Closed Sets in Topological Spaces / Journal of Xian Shiyou University, Natural since edition, ISSN:1673-064X.
25. Mishra S, V. Joshi, N. Bhardwaj, On Generalized Pre Regular Weakly( $gprw$ ) – Closed Sets in Topological Spaces, IMF, Vol. 7, 2012, No. 40, 1981-1992.
26. Mukundhan C, N. Nagaveni, A weaker form of a Generalized Closed Set, Int. J. Contemp. Maths. Sciences, Vol. 6, 2011, No. 20, 949-961.
27. Meena kumari N & T. Indira,  $r^*g^*$  - Closed Sets in Topological Spaces, www.researchmathsci.org, APAM: Vol. 6, No. 2, 2014, 125-132.
28. Nitin Bhardwaj, Harpreet Kaur, B. P. Garg, On Regular  $\beta$  - Generalized Closed Sets in Topological Spaces / GJPAM.ISSN 0973-1768 Volume 11, Number 2 (2015), pp. 875-886.
29. Norman Levine, Generalized Closed Sets in Topology, Rend Cir.Mat.Palamo 2(1970), 89 – 96.
30. Norman Levin, Semi-open and semi-continuity in topological spaces, Amer. Math. Monthly, 70 (1963) 36-41.

31. Palaniappan N and K. Chandra sekhar Rao / Regular Generalized Closed Sets/ Kyungpook Math.J(1993) 33, 211 – 219.
32. Parimelazhagan R and V. Subramonia Pillai, Strongly  $g^*$ - Closed Sets in Topological Spaces, Int.Journal of Math. Analysis, Vol.6, 2012, no .30, 1481 – 1489.
33. Pauline Mary Helen M,  $(gsp)^*$  - Closed Sets in Topological Spaces / International Journal of Mathematics trends and technology – volume 6 – February 2014.
34. Paulin Mary Helen M,  $(g^*p)^*$  - Closed Sets In Topological Spaces. International Journal of Mathematics Trends and Technology – Volume 6 – February 2014 ISSN: 2231-5373 <http://www.ijmttjournal.org> Page 87.
35. Ponmalar P, Between Strongly  $g$  - Closed Sets and Strongly  $g^{**}$  - Closed Sets in Topological Spaces 2019 JET 1R March 2019, Volume 6, Issue 3, [www.jetir.org](http://www.jetir.org) (ISSN – 2349 – 5162).
36. Sathishmohan P, Rajendran V, Chinnapparaj L and Radha K, On  $\theta g^*$ closed sets in topological spaces,Journal of Mathematics and Information, Vol 13,2018, 65-80, ISSN: 2349-0632 (P), 2349-0640(online).
37. Savithiri D and Janaki C, On Regular  $\wedge$  Generalized Closed Sets in Topological Spaces, International Journal of Mathematics Archive – 4(4), 2013, 162-169.
38. Sekar C and Rajakumari, On Alpha Generalized Star Pre-Closed Sets in Topological Spaces, IJMER.
39. Sekar S, G. Kumar, On  $gar$  - Closed Sets in Topological Spaces, IJPAM, Volume 108 No. 4 2016, 791-800.
40. Sekar S and S. Loganayagi, On Generalized  $b$  Star - Closed Set in Topological Spaces, Malaya J. Mat. 5(2)(2017) 401-406.
41. Shaini Melina, M Trinita Pricilla, On  $(g\alpha)^{**}$  - Closed Sets in Topological Spaces IJCRT /Volume 10, Issue 3 march 2022/ISSN:2320-2882.
42. Shyla Isac Mary T & G. Abirami  $\alpha(gg)^*$  - Closed Sets in Topological Spaces / IJMTT, 2022, 68(3), 5-10.
43. Sreeja D and S. Sasikala / Generalized Star Semi Regular Closed Sets in Topological Spaces / Malaya J.Mat S(1)(2015)42-56.
44. Velicko N V, H-Closed Topological Spaces, Amer.math.soc. Transl.78(1968), 103-116.
45. Vidhya D and R. Parimelazhagan,  $g^*b$  - Closed Sets in Topological Spaces, Int. J. Contemp. Math. Sciences, Vol. 7(2022), no. 27, 1305-1312.

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