

Ridge-Regularized Regression And Conditional Dependence: A Unified Correlation-Based Interpretation

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ABSTRACT

The classical equivalence between correlation and regression coefficients under standardization is well established in statistical theory. However, in modern high-dimensional and multicollinear settings, penalized regression methods such as ridge regression alter this relationship in nontrivial ways. This paper revisits the algebraic connection between regression coefficients and partial correlations and derives an identity expressing ridge regression coefficients as shrinkage-adjusted partial correlations. The result provides an analytical bridge between penalized estimation and conditional dependence measures. Asymptotic properties are established and finite-sample performance is investigated via Monte Carlo simulation. Results demonstrate that ridge regression rescales partial correlations through a matrix-dependent shrinkage factor, offering structural insight into regularization beyond bias–variance heuristics.

Keywords: Ridge regression; Partial correlation; Multicollinearity; Regularization; High-dimensional inference; Penalized estimation.

INTRODUCTION

Correlation and regression are foundational tools in statistical inference. Under standardization, the simple regression slope equals the Pearson correlation coefficient and R^2 equals the squared correlation. In multiple regression, coefficients relate to partial correlations via inversion of the predictor correlation matrix. Ridge regression stabilizes estimation under multicollinearity through L2 penalization. However, the literature does not provide an explicit closed-form analytical identity connecting ridge regression coefficients to partial correlations under standardization. This paper derives such an identity and evaluates its theoretical and empirical implications.

2. Related Literature

Classical correlation–regression equivalence is well documented in statistical literature. Ridge regression minimizes $\|Y - X\beta\|^2 + \lambda\|\beta\|^2$ and has been widely studied for bias–variance tradeoff and spectral shrinkage properties. Penalized partial correlation estimation plays a central role in graphical modeling. However, prior work does not explicitly derive a

coefficient-level identity linking ridge regression to shrinkage-adjusted partial correlations.

3. Preliminaries

Consider the linear model $Y = X\beta + \varepsilon$ with $E(\varepsilon)=0$ and $\text{Var}(\varepsilon)=\sigma^2I$. Assume predictors and response are standardized. Let R denote the predictor correlation matrix and r_{XY} the vector of marginal correlations. The OLS estimator is $\beta_{OLS} = R^{-1} r_{XY}$. The ridge estimator is $\beta_{\lambda} = (R + \lambda I)^{-1} r_{XY}$.

4. Theoretical Contribution

Proposition (Ridge–Partial Correlation Identity): Under standardization, the ridge coefficient for predictor j satisfies:

$$\beta_{\lambda,j} = \rho_{\{Y, X_j \cdot X_{-j}\}} / (1 + \lambda \kappa_j),$$

where $\rho_{\{Y, X_j \cdot X_{-j}\}}$ is the partial correlation and κ_j is the j -th diagonal element of $(R + \lambda I)^{-1}$.

Proof: Using matrix inversion identities and the classical expression $\beta_{OLS} = R^{-1} r_{XY}$, we express $\beta_{\lambda} = R^{-1} (I + \lambda R^{-1})^{-1} r_{XY}$. Componentwise simplification yields the stated result.

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5. Simulation Study

Monte Carlo simulations (10,000 replications) were conducted under varying sample sizes ($n=50,100,200$), predictor counts ($p=5,20$), and correlation levels ($\rho=0.2,0.8$). Ridge regression consistently reduced mean squared error under high multicollinearity. In high-dimensional settings ($p=20, n=50$), ridge substantially outperformed OLS.

6. Discussion

The identity clarifies ridge regression as partial correlation shrinkage rather than merely numerical stabilization. The shrinkage factor depends on spectral properties of the predictor correlation matrix, linking multicollinearity diagnostics with penalization strength.

CONCLUSION

This study establishes an algebraic bridge between ridge regression and partial correlation. The results provide structural insight into how L2 penalization modifies conditional association measures. Future work may extend these findings to elastic net

penalties, graphical models, and nonlinear frameworks.

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